Pick Up Graded Work

Sit in Alternate Seats & Rows

Sign In

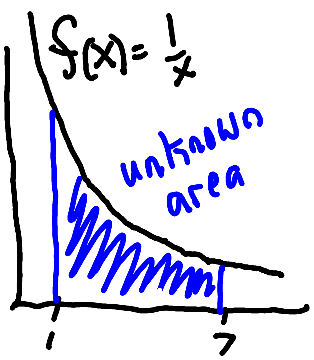
Today Section 5.3 The Evaluation Theorem; Quiz Q8

Tomorrow Recitation: Problems from 5.1, 5.2, 5.3

Recall Recent Stuff

Section 5.1, 5.2: $\underline{\text{Signed Area}} = SA = \lim_{n\to\infty} R_n = \int_{x=a}^{x=b} f(x)\,dx$

We saw an example involving $\underline{\text{estimating}}$ the $\underline{\text{value}}$ of the S.A.



$$= \underbrace{\int_{x=1}^{x=7} \frac{1}{x}\,dx}_{\text{Definite Integral is the exact value}} \approx 1.946$$

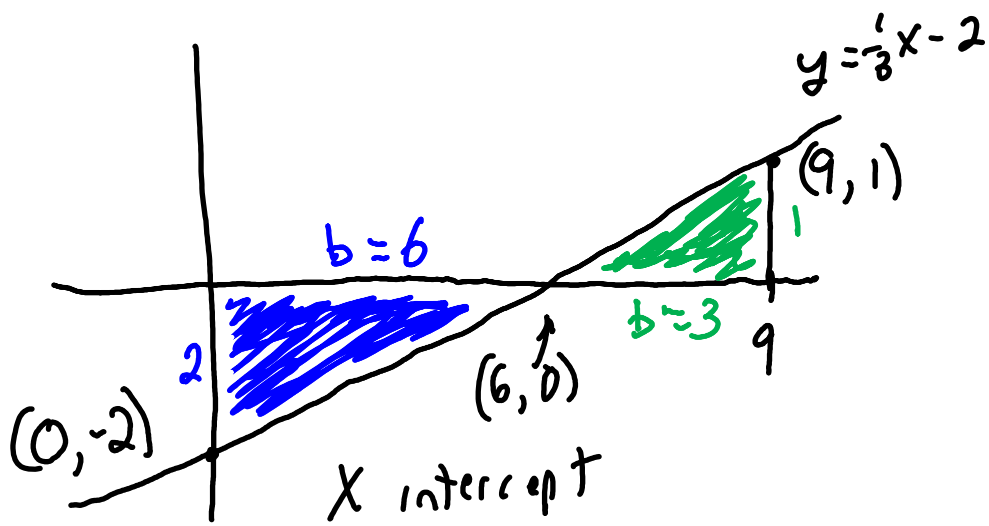
Definite Integral is the <u>exact</u> <u>value</u>

estimate obtained by using computer to compute L_n, R_n with higher & higher values of n.

In some cases, we can compute the definite integral <u>exactly</u> using geometry.

[Example 1] Find the exact value of $\int_0^9 \frac{1}{3}x - 2\,dx$ using geometry.

<u>Solution</u> That definite integral is the value of this <u>signed area</u>.



$$SA = -\text{Blue Area} + \text{Green Area}$$

$$= -\frac{1}{2}(6)\cdot 2 + \frac{1}{2}(3)1$$

$$= -6 + \frac{3}{2}$$

$$= -\frac{9}{2}$$

$$0 = \frac{1}{3}x - 2$$

$$2 = \frac{1}{3}x$$

$$6 = x$$

end of example

Review of ways we have found values for <u>definite integrals</u>

- Used <u>Computer</u> to <u>estimate</u> a value by computing L_n & R_n for higher & higher values of n.
- Used <u>geometry</u> to get <u>exact</u> value for function that had a simple, geometric graph.

<u>Obvious Question</u>: Is there a way to compute the <u>value</u> of the definite integral, $\lim_{n\to\infty} R_n$, <u>analytically</u>, <u>exactly</u>, for a general, curvy function?

<u>Great News</u> Yes! Book presents analytical techniques on p. 272-274.

<u>Bad News</u> Techniques are really hard. Above the level of MATH 2301.

<u>Question</u>. Is there an <u>easier way</u> to compute the value of a Definite Integral?

Good News: Yes!

The Evaluation Theorem (ET) (Section 5.3)

$$SA = \underbrace{\int_{x=a}^{x=b} f(x)\,dx}_{\text{this is officially defined to mean}} \underset{ET}{=} F(b) - F(a)$$

this is officially defined to mean

$$\lim_{n\to\infty} R_n$$

That's really hard to compute!
Above the level of MATH 2301

$\underbrace{F(x) \text{ is any antiderivative of } f(x)}_{\text{easy to compute!}}$

easy to compute!

Notice: Theorem expresses a relationship between
Definite Integrals and Antiderivatives

[Example 2] Use Evaluation Theorem to find the exact simplified value of the definite integral $\int_1^7 \frac{1}{x}\,dx$.

<u>Solution</u>

$$SA = \int_1^7 \frac{1}{x}\,dx \underset{\substack{\uparrow \\ ET}}{=} \left(\ln(|7|) + \cancel{C}\right) - \left(\ln(|1|) + \cancel{C}\right)$$

$a = 1$, $b = 7$

$f(x) = \frac{1}{x}$

$F(x) = \ln(|x|) + C$

$$= \ln(|7|) - \ln(|1|)$$

$$= \ln(7) - \ln(1)$$

$\ln(p) - \ln(q) = \ln\left(\frac{p}{q}\right)$

(Could also have more simply just used the fact that $\ln(1) = 0$.)

$$= \ln\left(\frac{7}{1}\right)$$

$$= \ln(7)$$ exact answer

$$\approx 1.9459$$ approx answer agrees with Friday's estimate of 1.946!!

[Example 3] Find exact value of $\int_0^9 \frac{1}{3}x - 2\,dx$ using the evaluation theorem.

Solution

$$SA = \int_0^9 \frac{1}{3}x - 2\,dx$$

$$\underset{ET}{=} \left(\frac{9^2}{6} - 2(9) + \cancel{C}\right) - \left(\underbrace{\frac{0^2}{6} - 2(0)}_{0} + \cancel{C}\right)$$

$$= \frac{81}{6} - 18$$

$$= \frac{27}{2} - \frac{36}{2}$$

$$= -\frac{9}{2}$$ exact answer, matches result of [Example 1]

$a = 0 \qquad b = 9$

$$f(x) = \frac{1}{3}x - 2$$

$$= \frac{1}{3}x^1 - 2 \cdot 1$$

$$= \frac{1}{3}x^1 - 2x^0$$

$n=1$ $n=0$

$$F(x) = \frac{1}{3}\frac{x^{1+1}}{1+1} - 2\frac{x^{0+1}}{0+1} + c$$

$$= \frac{x^2}{6} - 2x + c$$

End of Example

End of Lecture