Today: Finish Section 5.3 The Evaluation Theorem

Tomorrow Recitation: Problems from Section 5.3

Wed: Section 5.4 the Fundamental Theorem of Calculus

Fri: More of Section 5.4

Quiz Q9 over 5.3 and Wednesday Content of 5.4 ("Area Functions")

Meeting Part 1 The Net Change Theorem (Section 5.3)

Recall the

Evaluation Theorem (ET)

$$\underbrace{\int_a^b f(x)\,dx}_{\substack{\text{this is officially defined}\\ \text{to be a limit of sums}}} \underset{ET}{=} \underbrace{F(b) - F(a)}_{\text{where } F(x) \text{ is any antiderivative of } f(x)}$$

$$\lim_{n\to\infty} R_n$$

Extremely difficult calculation that we don't cover in 2301

Right side is straightforward

Since $F(x)$ is an antiderivative of $f(x)$,

we know that $F'(x) = f(x)$

Replace $f(x)$ with $F'(x)$ in the Evaluation Theorem

The result is a new theorem:

The Net Change Theorem (NCT)

$$\underbrace{\int_a^b F'(x)\,dx}_{\text{Definite Integral of the rate of change of } F(x)} \underset{NCT}{=} \underbrace{F(b) - F(a)}_{\text{Net change in } F(x) \text{ when } x \text{ changes from } a \text{ to } b}$$

[Example] (5.3#60, Similar to 5.3#59)

An object moves along a line with velocity $v(t) = t^2 - 2t - 8$ for $0 \le t \le 9$ where t is time in seconds, and $v(t)$ is the velocity at time t, in $\frac{\text{feet}}{\text{second}}$.

ⓐ Find the displacement over the time interval $[3, 6]$

Solution

displacement = change in position

$= s(6) - s(3)$

$\underset{NCT}{=} \int_3^6 s'(t)\,dt$

$$= \int_3^6 v(t)\,dt$$

$$= \int_3^6 \underbrace{t^2 - 2t - 8}_{v(t)}\,dt$$

$$\overset{=}{\scriptstyle ET} V(6) - V(3)$$ where $V(t)$ is any antiderivative of $v(t)$

Antiderivative Details

$v(t) = t^2 - 2t - 8$ rewrite for clarity

$$= t^2 - 2 \cdot t^1 - 8 \cdot t^0$$

$n=2$ $n=1$ $n=0$

antideriv

$$V(t) = \frac{t^{2+1}}{2+1} - 2\frac{t^{1+1}}{1+1} - 8\frac{t^{0+1}}{0+1}$$

$$= \frac{t^3}{3} - \frac{2t^2}{2} - \frac{8t}{1} = \frac{t^3}{3} - t^2 - 8t$$

$$= \left(\frac{(6)^3}{3} - (6)^2 - 8(6)\right) - \left(\frac{(3)^3}{3} - (3)^2 - 8(3)\right)$$

$$= \left(\frac{216}{3} - 36 - 48\right) - (9 - 9 - 24)$$

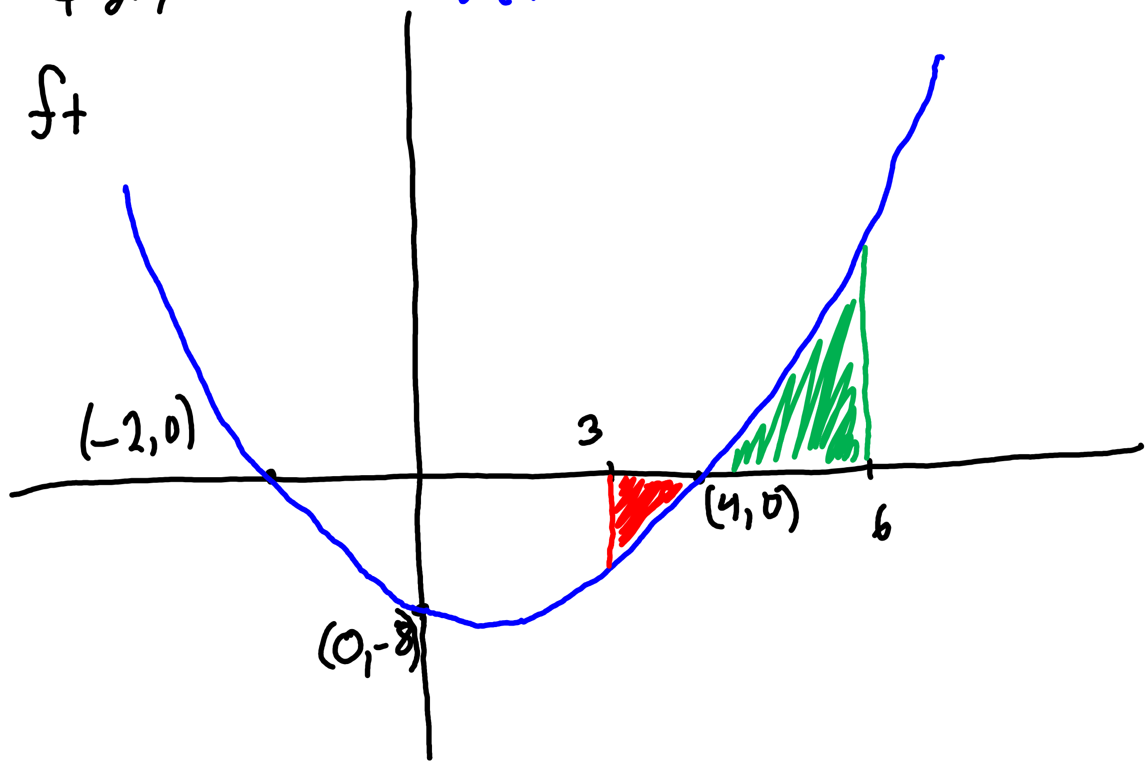
$= (72 - 36 - 48) - (-24)$ ⑤

$= (36 - 48) + 24$

$= -12 + 24$

$= 12$ ft

$v(t) = t^2 - 2t - 8 = (t+2)(t-4)$



displacement $= 12 =$ Green $-$ Red

(b) Find the <u>distance traveled</u> over the same interval $[3,6]$

<u>Solution</u>

$$\text{distance traveled} = \int_3^6 \text{speed}(t)\,dt$$

remember $\text{speed}(t) = |\text{velocity}(t)| = |v(t)|$

$$= \int_3^6 |v(t)|\,dt$$

remember

$$|v(t)| = \begin{cases} v(t) & \text{when } v(t) \text{ is pos} \\ -v(t) & \text{when } v(t) \text{ is neg} \end{cases}$$

$$= \int_3^4 -v(t)\,dt + \int_4^6 v(t)\,dt$$

on this interval, $v(t) \le 0$, so $|v(t)| = -v(t)$

on this interval, $v(t) \ge 0$, so $|v(t)| = v(t)$

$$= -\int_3^4 v(t)\,dt + \int_4^6 v(t)\,dt$$

$$= -\left[\left(\frac{(4)^3}{3} - (4)^2 - 8(4)\right) - \left(\frac{(3)^3}{3} - (3)^2 - 8(3)\right)\right] + \left[\left(\frac{(6)^3}{3} - 6^2 - 8(6)\right) - \left(\frac{(4)^3}{3} - (4)^2 - 8(4)\right)\right]$$

$$= -\left[\left(\frac{64}{3} - 16 - 32\right) - \left(-24\right)\right] + \left[\left(\frac{216}{3} - 36 - 48\right) - \left(\frac{64}{3} - 16 - 32\right)\right]$$

$$= -\left[\frac{64}{3} - 24\right] + \left[(72 - 36 - 48) - \left(\frac{64}{3} - 48\right)\right]$$

$$= -\frac{64}{3} + 24 + \left[(-12) - \frac{64}{3} + 48\right]$$

$$= -\frac{128}{3} + 60$$

$$= -\frac{128}{3} + \frac{180}{3}$$

$$= \frac{52}{3} \text{ ft} \approx 17.333 \text{ ft}$$

exact   decimal approx

[End of example]

Meeting Part 2 Indefinite Integrals

Definition of Indefinite Integral

Symbol: $\int f(x)\,dx$

Spoken: The indefinite integral of $f(x)$.

Meaning: The general antiderivative of $f(x)$

[Example]

$$\int x^2 - 2x - 8\,dx = \frac{x^3}{3} - x^2 - 8x + C$$

[Example]

$$\int \sec^2(x)\,dx = \tan(x) + C$$

Meeting Part 3 Evaluation Notation and Change in y notation

Recall

Definition of Evaluation Notation

Symbol: $f(x)\Big|_{x=a}$

Spoken: f evaluated at a

Meaning: $f(a)$

Definition Change in f(x) notation

Symbol: $f(x)\Big|_{a}^{b}$

Spoken: change in f(x) when x changes from a to b

Meaning: $f(b) - f(a)$

Meeting Part 4 Re-write the Evaluation Theorem Using the New Notation

$$\int_a^b f(x)\,dx \underset{ET}{=} F(b)-F(a) = F(x)\Big|_a^b = \left(\int f(x)\,dx\right)\Big|_a^b$$

New version of the Evaluation Theorem

$$\boxed{\int_a^b f(x)\,dx \underset{ET}{=} \left(\int f(x)\,dx\right)\Big|_a^b}$$

Example

$$\int_3^6 t^2-2t-8\,dt \underset{ET}{=} \left(\int t^2-2t-8\,dt\right)\Big|_3^6 = \left(\frac{t^3}{3}-t^2-8t+C\right)\Big|_3^6 = 12$$

(Example from earlier in the lecture)

End of Lecture