Sit in Pairs

Sign In

Today: Start Section 5.4

Friday (Dec 1): Finish discussing Section 5.4

Quiz Q9 Covers 5.3, start of 5.4

Thurs Dec 14

Final Exam

2:30pm - 4:30pm

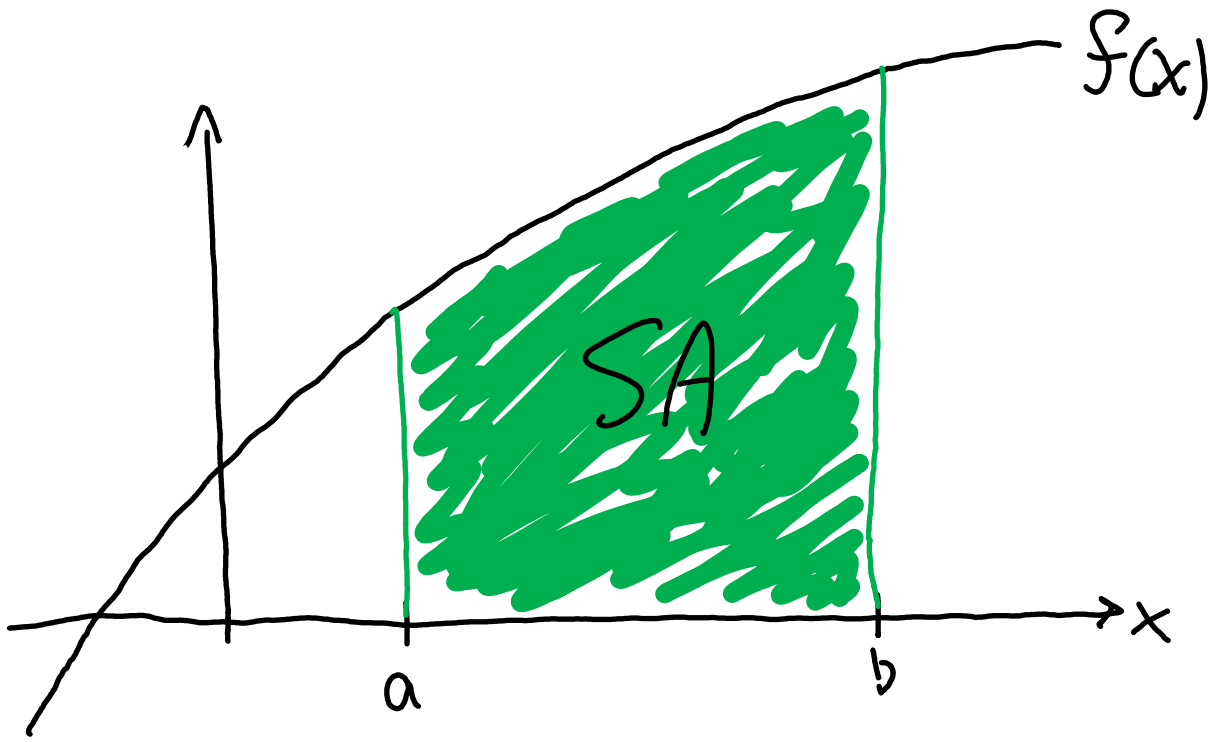
in various rooms in Morton Hall

(our room to be announced later)

Meeting Part 1 Area Functions (section 5.4)

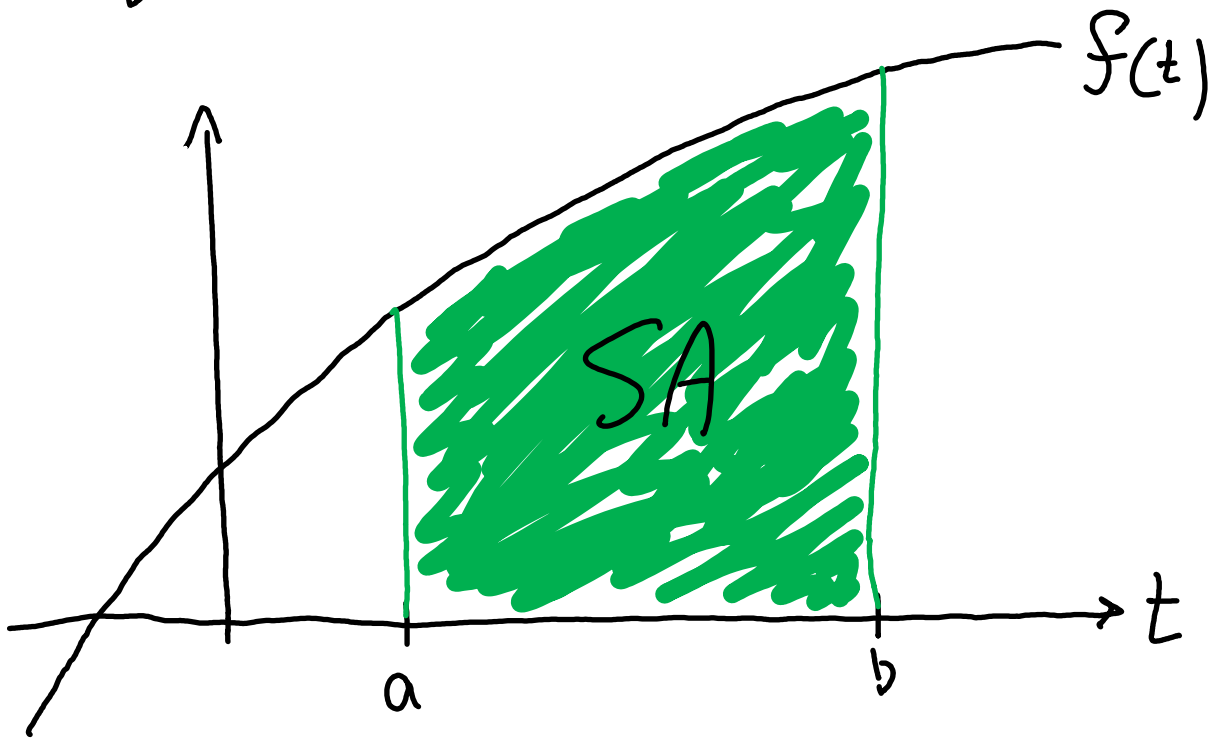
Recall that signed area is denoted by a definite integral

$$SA = \int_{x=a}^{x=b} f(x)\,dx$$



We could use a different variable

$$SA = \int_{t=a}^{t=b} f(t)\,dt$$



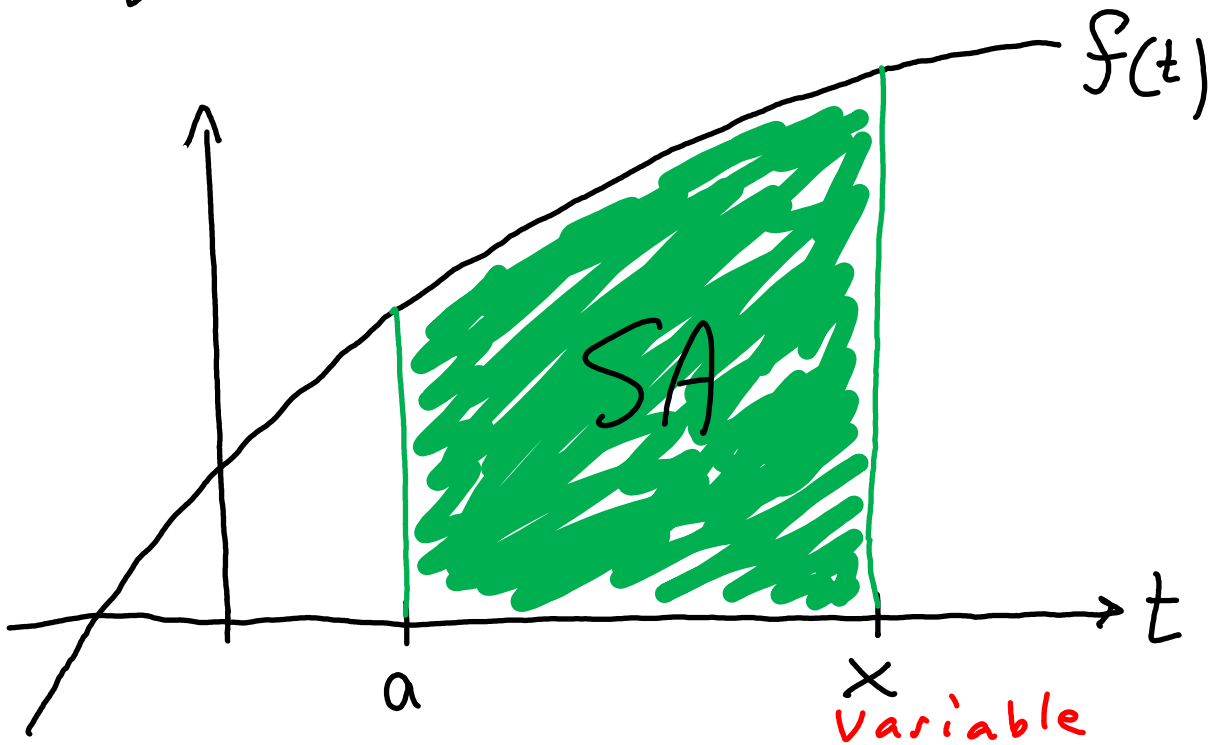
This definite integral is a <u>number</u>.
(The number does not depend on t.)

Replace the number b with a variable x

$$SA(x) = \int_{t=a}^{t=x} f(t)\,dt$$

$t=x$ ← variable

$t=a$ ← fixed number

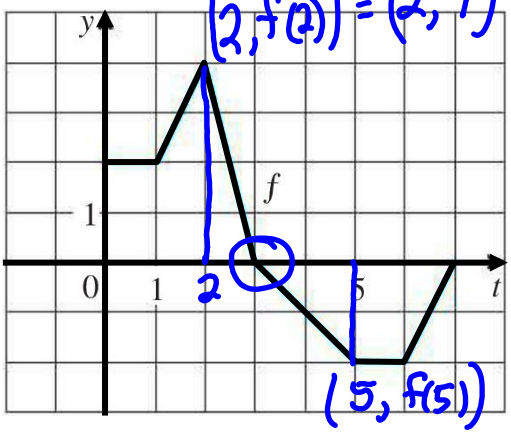


The signed area (the value of the definite integral) will depend on what value of x is used. So the signed area (the definite integral) is a function of x It is called an area function for f.

(5)

Class Drill: Area Function

The graph of $f(t)$ is shown at right.



(2, f(2)) = (2, 4)

(5, f(5)) = (5, -2)

The function $g(x)$ is defined by the equation

$$g(x) = \int_{t=0}^{t=x} f(t)dt$$

g is an *area function for* f

Part 1: The first goal is to build a graph of $g(x)$.
On the back of this page are eight copies of the graph of $f(t)$.

(a) Using these graphs, make eight pictures that illustrate the quantity

$$g(x) = \int_{t=0}^{t=x} f(t)dt$$

$g(2) = \int_{t=0}^{t=2} f(t)\,dt$

for the eight x values $x = 0,1,2,3,4,5,6,7$

(b) Using the pictures, find the value of $g(x)$ for $x = 0,1,2,3,4,5,6,7$ and write those values in the table at right.

x	$g(x)$
0	
1	
2	5
3	
4	
5	
6	
7	

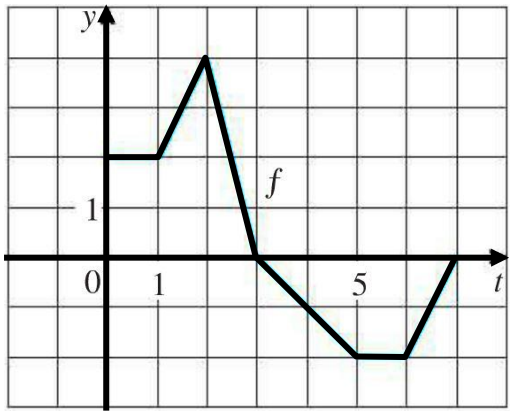
(c) Using the data from your table, make a graph of $g(x)$ on the interval $0 \le x \le 7$.

Part 2: Questions to be answered ***without*** using your graph from **Part 1**.

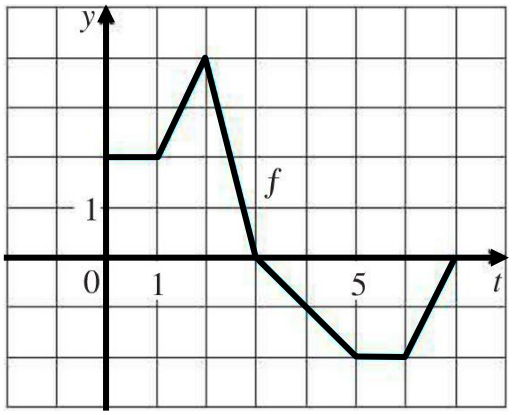
(d) What is the value of $g'(2)$? $g'(2) \underset{FTC1}{=} f(2) = 4$

(e) What is the value of $g'(5)$? $g'(5) \underset{FTC1}{=} f(5) = -2$

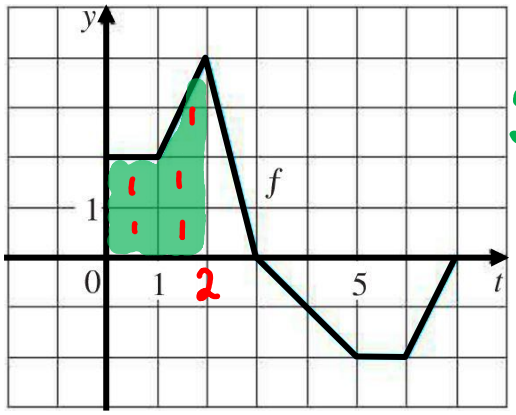
(e) At what x value does g have a local max? where g' changes from pos to neg.
That is, where f changes from pos to neg. This happens at $x = 3$!



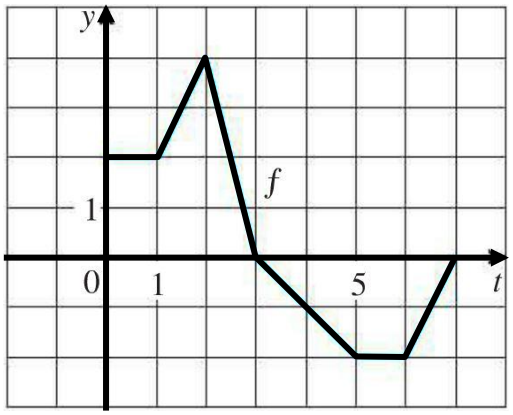
$g(0)=$



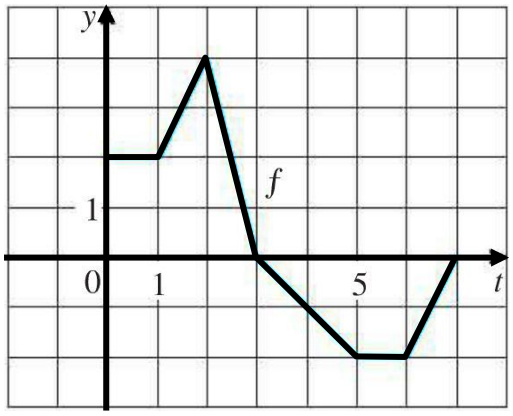
$g(1)=$



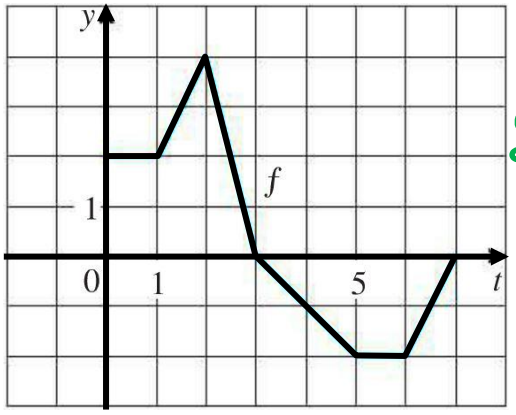
$g(2)=5$



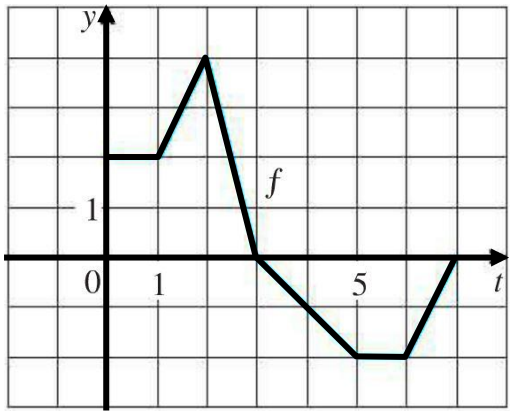
$g(3)=$



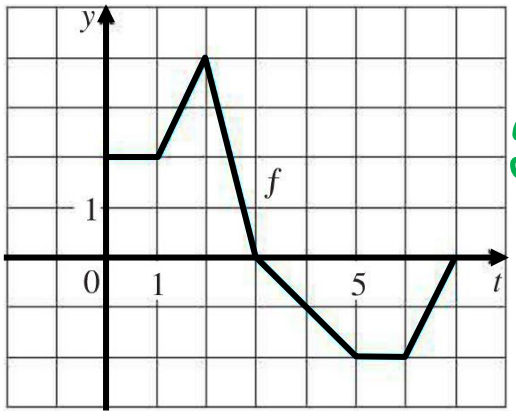
$g(4)=$



$g(5)=$

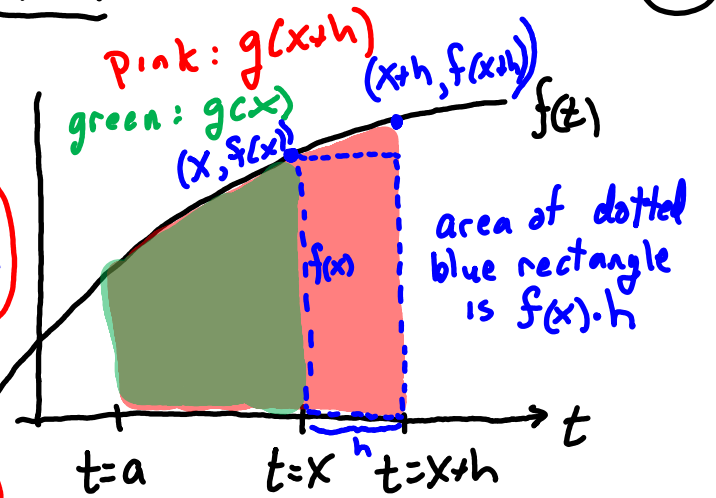


$g(6)=$



$g(7)=$

Meeting Part 2 Differentiating an Area Function



$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

Numerator is the uncovered pink region

Value of numerator is close to $f(x) \cdot h$

Value of the fraction is very close to $f(x)$

using the definition of the derivative

huge fact: the limit exists and has value $f(x)$.

$= f(x)$

This result is so important that it gets a very grand name.

The Fundamental Theorem of Calculus (FTC)

Part 1 $\frac{d}{dx}\left(\int_{t=a}^{t=x} f(t)\,dt\right) \underset{\text{FTC 1}}{=} f(x)$

The derivative of an area function is just the original function

Expresses relationship between Definite Integrals and Derivatives.

Part 2 is something that we've already studied. The Evaluation Theorem

$$\int_{x=a}^{x=b} f(x)\,dx \underset{\substack{\text{ET}\\ \text{FTC 2}}}{=} \left(\int f(x)\,dx\right)\Bigg|_{x=a}^{x=b} = F(b) - F(a)$$

Expresses relationship between Definite Integrals and Indefinite Integrals (Antiderivatives)

[Example] of using the FTC.

Let $g(x) = \int_{t=5}^{t=x} (t - t^2)^8 \, dt$

$f(t) = (t - t^2)^8$

$f(\ ) = (\ ) - (\ )^2)^8$ empty version

$f(x) = (x - x^2)^8$

Find $g'(x)$

Solution

$g'(x) \underset{\text{FTC 1}}{=} f(x) = (x - x^2)^8$

end of example

End of Meeting