MATH 2301 (Barsamian) Lecture #34, Fri Dec 1, 2023

Pick Up Graded Work

Sit in Alternate Seats + Rows

Sign In

Today: Finish Section 5.4 The Fundamental Theorem of Calculus
Quiz Q9

Next Week: Section 5.5 The Substitution Method
Review Day

Thu Dec 14: Final Exam 2:30pm - 4:30pm (More info next week)

Finishing Section 5.4 The Fundamental Theorem of Calculus

Meeting Part 1: Two more examples involving the Fundamental Theorem

[Example] Let $g(x) = \int_0^x \sqrt[3]{1+r^5}\,dr$ Find $g'(x)$

Solution

$g'(x) \underset{\text{FTC 1}}{=} f(x) = \sqrt[3]{1+x^5}$

$f(r) = \sqrt[3]{1+r^5}$

$f(\ \ ) = \sqrt[3]{1+(\ \ )^5}$ empty version

$f(x) = \sqrt[3]{1+x^5}$

Play with the notation of the fundamental theorem

If $g(x) = \int_a^x f(t)\,dt$ then $g'(x) = f(x)$

Change variable to z

If $g(z) = \int_a^z f(t)\,dt$ then $g'(z) = f(z)$

Empty version

If $g(\ ) = \int_a^{(\ )} f(t)\,dt$ then $g'(\ ) = f(\ )$

There are other ways of notating this

$$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) \underset{FTC1}{=} f(x)$$

Switch to prime notation

$$\left(\int_a^x f(t)\,dt\right)' \underset{FTC1}{=} f(x)$$

empty version

$$\left(\int_a^{(\ )} f(t)\,dt\right)' \underset{FTC1}{=} f(\ )$$ empty version

[Example] Let $h(x) = \int_0^{(x^4)} \sqrt[3]{1+r^5}\,dr$ Find $h'(x)$

Solution

Realize that $h(x)$ is a nested function

$h'(x) = \frac{d}{dx}\,\text{outer}(\text{inner}(x))$

ChainRule

$= \text{outer}'(\text{inner}(x)) \cdot \text{inner}'(x)$

$= \sqrt[3]{1+(x^4)^5} \cdot 4x^3$

$= \sqrt[3]{1+x^{20}} \cdot 4x^3$

Chain Rule Details

$\text{inner}(x) = x^4$

$\text{inner}'(x) = 4x^3$

$\text{outer}(\ ) = \int_0^{(\ )} \sqrt[3]{1+r^5}\,dr$

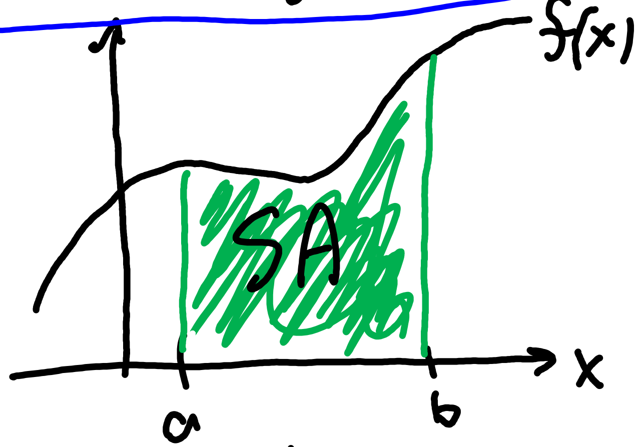
$\text{outer}'(\ ) = \left(\int_0^{(\ )} \sqrt[3]{1+r^5}\,dr\right)'$

$= \sqrt[3]{1+(\ )^5}$

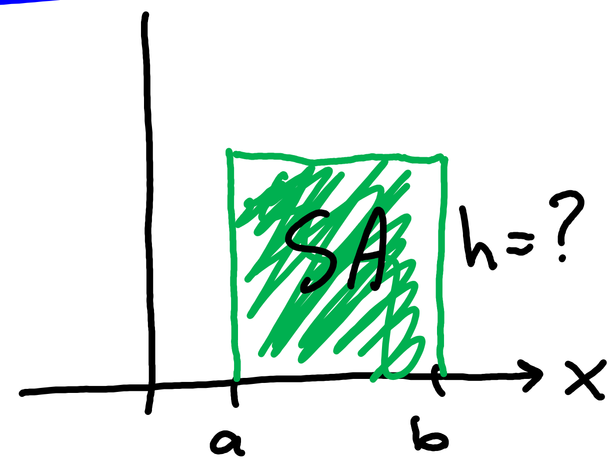
$(x^p)^q = x^{p \cdot q}$

Meeting Part 2 The Average Value of a Function on an Interval

Geometric Question Given a function f that is continuous on a given interval $[a,b]$, how tall would a rectangle parked on that same interval need to be in order to enclose the same signed area?



=



$$SA = \int_a^b f(x)\,dx \quad = \quad SA = (b-a)\cdot h$$

Divide both sides by $b-a$ in order to get h

answer $h = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

Answers our Question

This result is enshrined in a definition

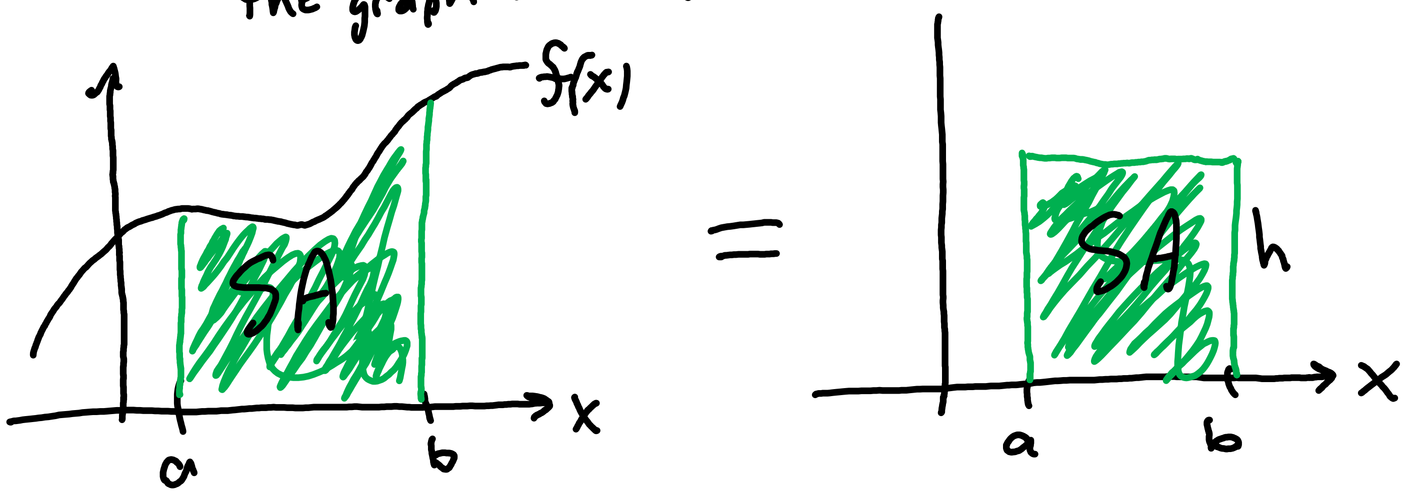
Definition The Average Value of a Function on an Interval

words: The average value of f on the interval $[a,b]$

usage: f is a function that is continuous on $[a,b]$

meaning: the number $h = \frac{1}{b-a}\int_a^b f(x)\,dx$

graphical significance: A rectangle of height h, parked on the same interval $[a,b]$, will have the same area as the region between the graph of $f(x)$ and the x axis on that interval



Example Find average value of $\frac{1}{\sqrt{x}}$ on the interval $[1,4]$

Solution

$$h = \frac{1}{4-1}\int_1^4 \frac{1}{\sqrt{x}}\,dx$$

$$\underset{FTC}{=} \frac{1}{3}\left(\int \frac{1}{\sqrt{x}}\,dx\right)\Bigg|_1^4$$

$$= \frac{1}{3}\left(2\sqrt{x} + C\right)\Bigg|_1^4$$

$$= \frac{1}{3}\left(\left(2\sqrt{4} + \cancel{C}\right) - \left(2\sqrt{1} + \cancel{C}\right)\right)$$

$$= \frac{1}{3}\left(4 - 2\right)$$

$$= \frac{2}{3}$$

Indefinite Integral Details

$n = -\frac{1}{2}$

$$\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx$$

$$= \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{x^{1/2}}{1/2} + C$$

$$= 2x^{1/2} + C$$

$$= 2\sqrt{x} + C$$

End of Example and End of Lecture