MATH 2301 (Barsamian) Lecture #37 Fri Dec 8, 2023

Pick up graded work

Sign In

Today: Review

Tue: \
Wed: } Isaac + Kenny will run review sessions. Room + time tba

Thu: Final Exam 2:30-4:30 room tba

Review

[1] find the limit two ways $\lim_{h \to 0} \frac{(-5+h)^2 - 25}{h}$

Solution #1 Do the limit

$$\lim_{h \to 0} \frac{(-5+h)^2 - 25}{h} = \lim_{h \to 0} \frac{(25 - 10h + h^2) - 25}{h}$$

$$= \lim_{h \to 0} \frac{-10h + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(-10 + h)}{h}$$

indeterminate form

Since $h \to 0$, we know $h \neq 0$, so we can cancel $\frac{h}{h}$

$$= \lim_{h \to 0} -10 + h$$

no longer indeterminate

$$= -10 + (0)$$

$$= -10$$

Second Solution

Recognize that $\lim_{h\to 0} \frac{(-5+h)^2-25}{h}$ is a

derivative computation $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

Solution strategy

identify $f(x)$

identify a

find $f'(x)$ using (easy) derivative rules

Sub in $x=a$ to get $f'(a)$

identify $a=-5$

identify $f(\quad)=(\quad)^2$ empty version. (the thing holding $a+h$)

$f(x) = x^2$

$f'(x) = 2x$ using (easy) derivative rules

$f'(a) = 2(-5) = -10$ Same answer as first solution

[2] Let $f(x) = \dfrac{1}{x^3 - 1}$

find (a) $\lim\limits_{x \to 1^-} f(x)$, (b) $\lim\limits_{x \to 1^+} f(x)$, (c) $\lim\limits_{x \to 1} f(x)$

Solution

(a) $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \dfrac{1}{x^3 - 1}$

x slightly less than 1

So x^3 will be slightly less than 1

So the denominator will be close to 0 and negative

So the ratio will be huge, negative

One solution method

x	$y = \frac{1}{x^3 - 1}$
0.9	$\frac{1}{(.9)^3 - 1}$
0.99	$\frac{1}{(.99)^3 - 1}$
0.999	$\frac{1}{(.999)^3 - 1}$

So $\lim\limits_{x \to 1^-} f(x) = -\infty$

(b) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{1}{x^3-1}$

x slightly greater than 1

x^3 will also be slightly greater than 1

So denominator will be close to 0 and positive

ratio will be huge, positive

Conclude $\lim_{x\to 1^+} f(x) = \infty$

(c) $\lim_{x\to 1} f(x) =$ Does not exist because left & right limits dont match.

~~Eddie says $\lim_{x\to 1} f(x) = \frac{1}{1^3-1} = \frac{1}{0}$ Does not exist~~

right answer but invalid reasoning

~~Freddie says $\lim_{x\to 1} f(x) = \frac{1}{1^3-1} = \frac{1}{0} = 0$~~

wrong answer

[3] $g(x) = \sqrt{9-x}$ Find $g'(x)$ two ways

(a) Derivative Rules

(b) Definition of derivative

Solution

(a) $g'(x) = \frac{d}{dx}(9-x)^{1/2}$

$\underset{\text{chain rule}}{=} \frac{1}{2\sqrt{9-x}} \cdot -1$

$= -\frac{1}{2\sqrt{9-x}}$

Chain Rule

inner(x) = 9 − x

inner′(x) = −1

outer( ) = ( )$^{1/2}$

outer′( ) $= \frac{1}{2}(\;)^{\frac{1}{2}-1}$

$= \frac{1}{2}(\;)^{-\frac{1}{2}}$

$= \frac{1}{2}\frac{1}{(\;)^{1/2}}$

$= \frac{1}{2\sqrt{(\;)}}$

(b) $g'(x) = \lim_{h \to} \frac{g(x+h)-g(x)}{h}$ indeterminate

$g(x) = \sqrt{9-x}$

$g(\ \ ) = \sqrt{9-(\ \ )}$ empty version

$g(x+h) = \sqrt{9-(x+h)}$

$= \sqrt{9-x-h}$

$= \lim_{h \to 0} \frac{\sqrt{9-x-h} - \sqrt{9-x}}{h}$

$= \lim_{h \to 0} \frac{\sqrt{9-x-h} - \sqrt{9-x}}{h} \cdot \frac{\sqrt{9-x-h} + \sqrt{9-x}}{\sqrt{9-x-h} + \sqrt{9-x}}$

trick

$(a-b)(a+b) = a^2 - b^2$

$= \lim_{h \to 0} \frac{(9-x-h) - (9-x)}{h(\sqrt{9-x-h} + \sqrt{9-x})}$

$= \lim_{h \to 0} \frac{-h}{h(\sqrt{9-x-h} + \sqrt{9-x})}$ Still indeterminate

Since $h \to 0$, we know $h \neq 0$, so we can cancel

$= \lim_{h \to 0} -\frac{1}{\sqrt{9-x-h} + \sqrt{9-x}}$ no longer indeterminate

$= -\frac{1}{\sqrt{9-x-(0)} + \sqrt{9-x}} = -\frac{1}{\sqrt{9-x} + \sqrt{9-x}} = \boxed{-\frac{1}{2\sqrt{9-x}}}$

Same answer as (a)

[4] Find derivative of $y = \dfrac{x^2 + 4\sqrt{x} + 3}{x^{3/2}}$

Quotient Rule would work but would be killer

Could rewrite and use Product Rule. Also killer.

Smarter rewrite into power function form

$$y = \frac{x^2}{x^{3/2}} + \frac{4x^{1/2}}{x^{3/2}} + \frac{3}{x^{3/2}} = x^{1/2} + 4x^{-1} + 3x^{-3/2}$$

$$y' = \frac{1}{2}x^{1/2-1} + 4(-1)x^{-1-1} + 3\left(-\frac{3}{2}\right)x^{-3/2-1}$$

$$= \frac{1}{2}x^{-1/2} - 4x^{-2} - \frac{9}{2}x^{-5/2}$$

$$= \frac{1}{2\sqrt{x}} - \frac{4}{x^2} - \frac{9}{2x^{5/2}}$$

[5] Find $F(x) = \int \frac{x^2 + 4\sqrt{x} + 3}{x^{3/2}} \, dx$

Solution there are no quotient or product rules!

No choice but to rewrite the integrand

$$F(x) = \int x^{1/2} + \frac{4}{x} + 3x^{-3/2} \, dx$$

$$= \frac{x^{1/2 + 1}}{1/2 + 1} + 4 \ln|x| + 3 \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C$$

[6] $f(x) = x^2 e^{(x)}$

ⓐ find $f'(x)$

ⓑ find $f'(0)$

ⓒ find $f'(1)$

ⓐ $f'(x) = \frac{d}{dx}\, \underbrace{x^2 e^{(x)}}_{\text{Product!!!}} \underset{\text{Product rule}}{=} (2x)e^x + x^2(e^x)$

$= (2x + x^2)e^{(x)}$

ⓑ $f'(0) = (2(0) + (0)^2)\, e^{(0)} = 0 \cdot 1 = 0$

famous value

ⓒ $f'(1) = (2(1) + (1)^2)\, e^{(1)} = 3e$

famous value

[7] $f(x) = x^2 \ln(x)$

(a) find $f'(x)$

(b) find $f'(1)$

(c) find $f'(e)$

Solution

(a) $f'(x) = \frac{d}{dx} \underbrace{x^2 \ln(x)}_{\text{Product!!}} \underset{\text{product rule}}{=} (2x)\ln(x) + x^2\left(\frac{1}{x}\right)$

$\underset{\text{simplify}}{=} 2x\ln(x) + x \underset{\text{simplify more}}{=} x(2\ln(x)+1)$

(b) $f'(1) = (1)(2\ln(1)+1) = 1(2\cdot 0 + 1) = 1$

famous value

(c) $f'(e) = (e)(2\ln(e)+1) = e(2\cdot 1 + 1) = e(3) = 3e$

important: $e \cdot 3$, not e^3

end of lecture