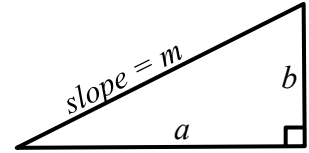


First Class Drill on Newton's Method Part 1: The Idea Behind Newton's Method

(a) In the triangle shown, find an equation for the slope m of the hypotenuse in terms of the lengths a and b .

$m =$



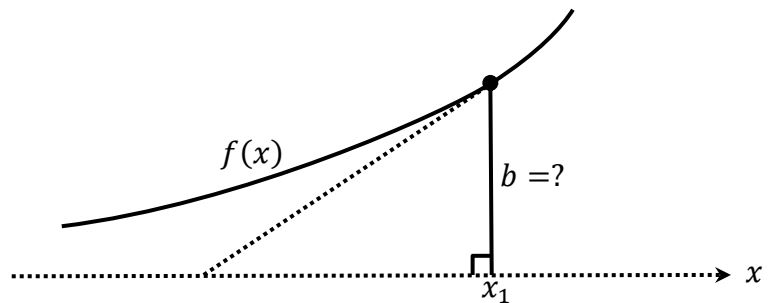
(b) Solve the equation for a in terms of m and b :

$a =$

(c) In the triangle shown, the upper right vertex lies on the graph of f .

How tall is the right leg?

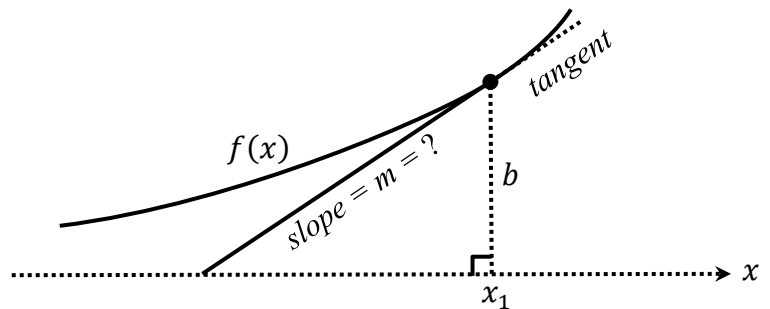
$b =$



(d) Suppose that the hypotenuse of the triangle is known to lie on the line that's tangent to the graph of f at the point where $x = x_1$

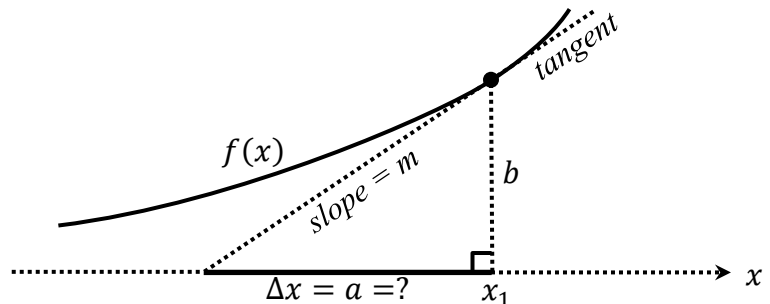
What is the hypotenuse slope m ?

$m =$



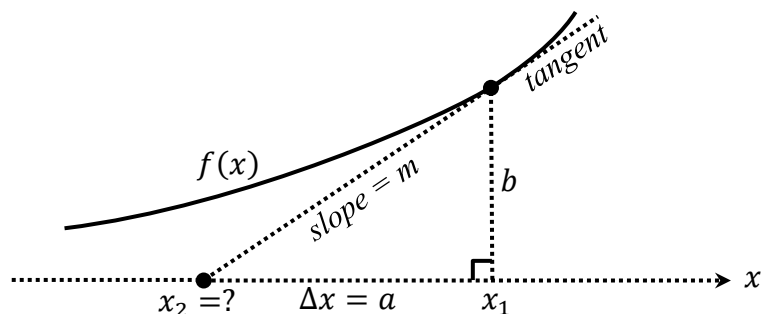
(e) For the same triangle, what is the base Δx ?

$\Delta x = a =$



(f) For the same triangle, what is the x coordinate x_2 ?

$x_2 =$



The Class Drill continues on the next page →

First Class Drill on Newton's Method Part 2: Using Newton's Method

Newton's Method

Given: A function f that is differentiable on an interval I and that has a root in I . That is, it is known that there exists a number r somewhere in I such that $f(r) = 0$.

Goal: Find an approximate value for the root r , accurate to d decimal places.

Step 1: Choose a value x_1 as an initial approximation of the root. (This is often done by looking at a graph.)

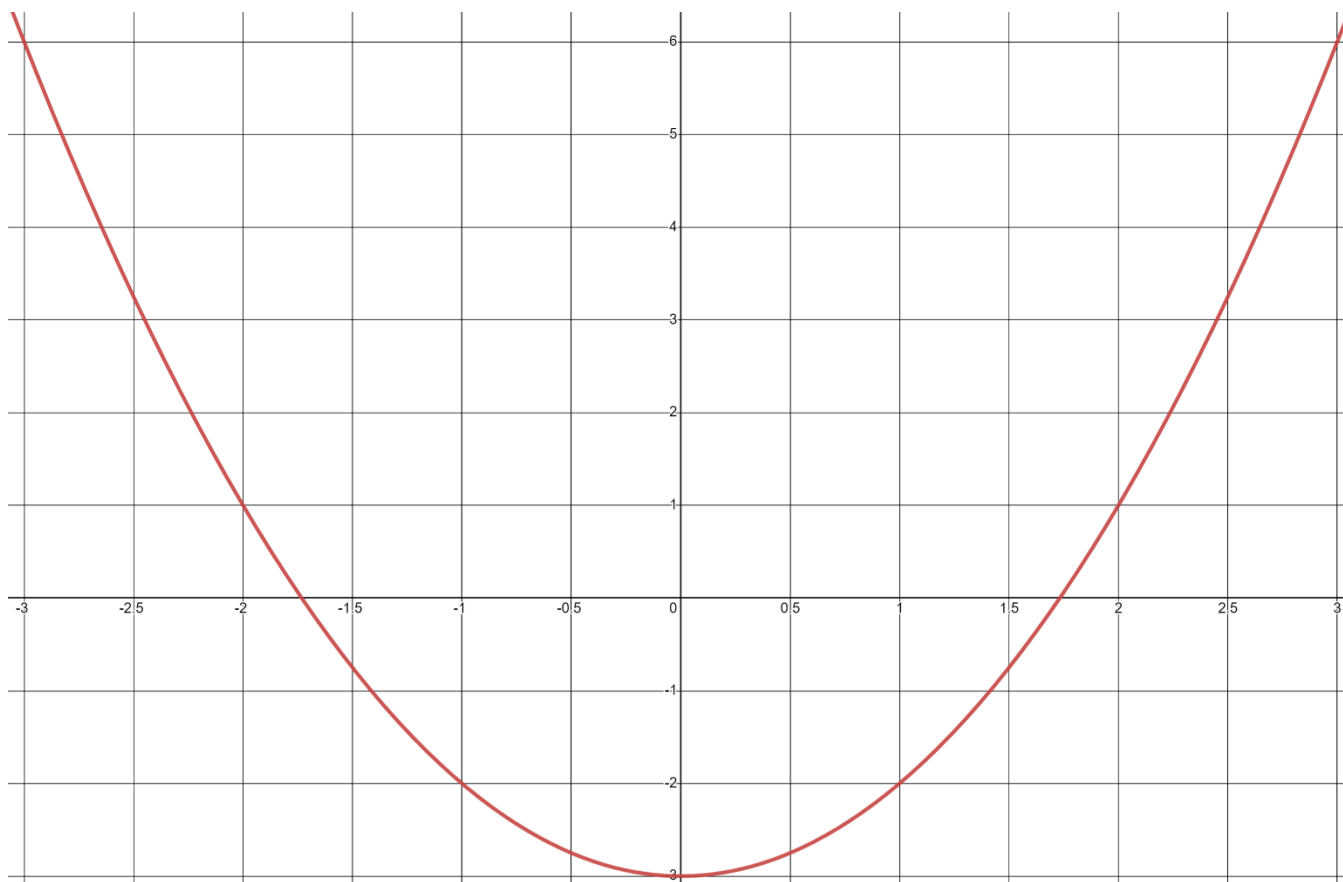
Step 2: Create successive approximations iteratively, as follows:

Given an approximation x_n , compute the next approximation x_{n+1} by using the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Step 3: Stop the iterations when successive approximations do not differ in the first d places after the decimal point. The last x value computed is the approximation of r .

Let $f(x) = x^2 - 3$. Observe that the graph of $f(x)$ shows an x intercept somewhere between $x = 1$ and $x = 2$. Using the terminology of roots, we would say that there is a root of f , that is, a number r such that $f(r) = 0$, and that r is somewhere between 1 and 2.



The goal is to use Newton's method to find an approximation for the root r . You will do the first three iterations only, using the initial approximation $x_1 = 3$. That is, you will find x_2, x_3, x_4 .

For the function $f(x) = x^2 - 3$,

(a) Compute $f'(x)$

(b) Fill out the following table. (Do the details below.)

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
1	$x_1 = 3$			$x_2 =$
2	$x_2 =$			$x_3 =$
3	$x_3 =$			$x_4 =$
4	$x_4 =$			

The Class Drill continues on the next page →

(C) A zoomed-in graph of $f(x)$ is shown below. You'll illustrate some of your results on this graph.

- Put a point at $(x_1, 0)$
- Put a point at $(x_1, f(x_1))$
- Draw the segment that connects $(x_1, 0)$ and $(x_1, f(x_1))$. This segment should be vertical.
- Put a point at $(x_2, 0)$.
- Draw the segment that passes through $(x_1, f(x_1))$ and $(x_2, 0)$. This segment should appear to be tangent to the graph of $f(x)$ at the point $(x_1, f(x_1))$.
- Put a point at $(x_2, f(x_2))$
- Draw the segment that connects $(x_2, 0)$ and $(x_2, f(x_2))$. This segment should be vertical.
- Put a point at $(x_3, 0)$.
- Draw the segment that passes through $(x_2, f(x_2))$ and $(x_3, 0)$. This segment should appear to be tangent to the graph of $f(x)$ at the point $(x_2, f(x_2))$.

