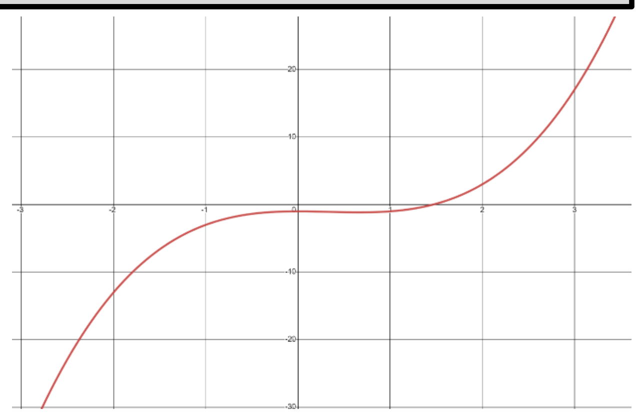
Second Class Drill on Using Newton's Method

Newton's Method
Given: A function f that is differentiable on an interval I and that has a root in I. That is, it is known that there exists a number r somewhere in I such that $f(r) = 0$.
Goal: Find an approximate value for the root r, accurate to d decimal places.
Step 1: Choose a value x_1 as an initial approximation of the root. (This is often done by looking at a graph.)
Step 2: Create successive approximations iteratively, as follows:
Given an approximation x_n, compute the next approximation x_{n+1} by using the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Step 3: Stop the iterations when successive approximations do not differ in the first d places after the decimal point. The last x value computed is the approximation of r.



Let $f(x) = x^3 - x^2 - 1$. Observe that the graph of $f(x)$ shows an x intercept somewhere between $x = 1$ and $x = 2$. Using the terminology of roots, we would say that there is a root of f, that is, a number r such that $f(r) = 0$, and that r is somewhere between 1 and 2.

The goal is to use Newton's method to find an approximation for the root r. You will do the first two iterations only, using the initial approximation $x_1 = 1$. That is, you will find x_2 and x_3.

For the function $f(x) = x^3 - x^2 - 1$,

(a) Compute $f'(x)$

(b) Fill out the following table. (Do the details on scrap paper.)

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
1	$x_1 = 1$			$x_2 =$
2	$x_2 =$			$x_3 =$
3	$x_3 =$			

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(C) A zoomed-in graph of $f(x)$ is shown below. You'll illustrate some of your results on this graph.

- Put a point at $(x_1, 0)$
- Put a point at $(x_1, f(x_1))$
- Draw the segment that connects $(x_1, 0)$ and $(x_1, f(x_1))$. This segment should be vertical.
- Put a point at $(x_2, 0)$.
- Draw the segment that passes through $(x_1, f(x_1))$ and $(x_2, 0)$. This segment should appear to be tangent to the graph of $f(x)$ at the point $(x_1, f(x_1))$.
- Put a point at $(x_2, f(x_2))$
- Draw the segment that connects $(x_2, 0)$ and $(x_2, f(x_2))$. This segment should be vertical.
- Put a point at $(x_3, 0)$.
- Draw the segment that passes through $(x_2, f(x_2))$ and $(x_3, 0)$. This segment should appear to be tangent to the graph of $f(x)$ at the point $(x_2, f(x_2))$.