Common Missed Opportunity: Cancel Common Factors

Cancel early, as soon as the common factors appear. Otherwise, you will end up having to rewrite common factors on subsequent steps. This takes time, clutters your work, and increases the chance of making a mistake.

In particular, **CANCEL BEFORE MULTIPLYING!! (CBM)** Otherwise, your multiplication step will be harder, and your simplifying will be harder, too!

The next five pages show samples of student work from a past quiz. The problem is a related rates problem involving a right triangle. The calculation uses the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

$$\frac{d}{dt}(a^2 + b^2) = \frac{d}{dt} c^2$$

$$a^2 \frac{1}{dt} + b^2 \frac{d}{dt} = c^2 \frac{1}{dt}$$

$$2(a)a' + 2(b)b' = 2(c)c'$$

$$2(5)1 + 2(12)b' = 2(13)0$$

$$10 + 24b' = 0$$

$$\frac{24b}{24} = -10$$

$$b' = -\frac{10}{24}$$

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$$a^2 + b^2 = c^2$$

$$\frac{d}{dx}a^2 + \frac{d}{dx}b^2 = \frac{d}{dx}c^2$$

$$2aa' + 2bb' = 2cc'$$

Chain rule for all 3

example!

$$\frac{d}{dx}(a)^2 = 2aa'$$

rearrange

$$2bb' = 2cc' - 2aa'$$

Plug-in

$$2(12)b' = 2(13)(0) - 2(5)(1 \text{ ft/sec})$$

Solve for b'

$$24b' = 0 - 10 \text{ ft/sec}$$

$$b' = \frac{10}{24} \text{ ft/sec}$$

$$\boxed{b' = -\frac{5}{12} \text{ ft/sec}}$$

$a = 5$

$c = 13$

$b = 12$

$a' = 1$

$c' = 0$

$a' = 1$

Ladder is sliding down the wall at

$\frac{5}{12}$ ft/sec

$\left[a^2 + b^2 = c^2\right] \frac{d}{dt}$

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chain rule

$2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dt} = 2c \cdot \frac{dc}{dt}$

we know c is a constant (13), so $\frac{dc}{dt} = 0$

plug in:

$2(5) \cdot 1 + 2(12) \frac{db}{dt} = 0$

$10 + 24 \frac{db}{dt} = 0$

$24 \frac{db}{dt} = -10$

$\frac{db}{dt} = \frac{-10}{24} = \boxed{-\frac{5}{12} \text{ ft/sec}}$

$b^2 = c^2$

$b = \sqrt{16}$

$b = \sqrt{1}$

$b = 1$

-25 -25 $b = 12$

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$\left(\frac{d}{dt}\right) a^2 + b^2 = c^2 \frac{d}{dt}$

c is a constant so $c' = 0$

$2aa' + 2bb' = 2cc'$

$2aa' + 2bb' = 0$

$\frac{2bb'}{2b} = \frac{-2aa'}{2b}$

$b' = \frac{-2aa'}{2b}$

$b' = \frac{-2(5ft)(-1\ ft/sec)}{2(12ft)} = \frac{10\ ft^2/sec}{24\ ft} = \frac{5}{12}\ \frac{ft}{sec}$

The top of the ladder is sliding down the wall at the instant when the foot of the ladder is 5ft from the wall at a rate $\frac{5}{12}\ \frac{ft}{sec}$

The Quiz continues on back ➔

$a^2 + b^2 = c^2$

$\frac{d}{dx}(a^2) + \frac{d}{dx}(b^2) = \frac{d}{dx}(c^2)$

$2aa' + 2bb' = 2cc'$

$2bb' = 2cc' - 2aa'$

$b' = \frac{2cc'}{2b} - \frac{2aa'}{2b}$

$b' = \frac{2(13)(0)}{2(12)} - \frac{2(5)(-1)}{2(12)}$

$b' = -\frac{2(5)}{2(12)}$

$b' = -\frac{10}{24}$

$b' = -\frac{5}{12}$ ft/sec

In all five of these samples, the student did end up with the right answer for their calculation. But you can see that they made their calculations more cluttered and more difficult by not cancelling the factor of 2 at the earliest opportunity.

And they all had to eventually cancel the 2 in the simplifying, anyway. (Except for the one student who did not simplify and who lost a point for not simplifying!)

(Not shown: There were many more quiz papers with this same error. I only included the samples where the student did end up with the right answer.)