Handout: Implicit Differentiation

Part 1: Equations – vs – Functions

[Example 1]

- $x^3 + y^3 = 7$ equation involving x and y describes y *implicitly*
- $y = (7 - x^3)^{1/3}$ equation involving x and y, solved for y in terms of x. Gives y as a function of x. Describes y *explicitly*.

Observe that the two equations above express the same relationship between x and y.

[Example 2]

- $x^2 + y^2 = 7$ equation involving x and y describes y *implicitly*. Expresses a relationship between x and y. Cannot be solved for y as a function of x. We know this because the graph is a circle. Fails the vertical line test. Can't be the graph of a function.

Part 2: Implicit Differentiation

Suppose we have equation involving x and y and we want to find $\frac{dy}{dx}$.

If the equation can be solved for y in terms of x. We should do that first

$$y = \text{some expression involving } x$$

then take the ordinary derivative

$$\frac{dy}{dx} = \frac{d}{dx}(\text{some expression involving x})$$

If the equation *cannot* be solved for y in terms of x, we can still find $\frac{dy}{dx}$ using a method called ***Implicit Differentiation***.

The Method of Implicit Differentiation

(Used for finding y' when x, y are related by an equation that is not solved for y.)

Starting with: An equation involving x and y.

Step 1: Take derivative of left and right sides of this new equation with respect to x. Keep in mind the difference between taking the derivative of x and taking the derivative of y.

$$\frac{d}{dx}x \underset{\substack{power\\rule\\with\ n=1}}{=} 1$$

$$\frac{d}{dx}y = y' \quad \text{This is unknown! We cannot go any farther.}$$

The result will be a new equation involving x and y and y'.

Step 2: Solve for y'. The result will be a new equation of the form

$$y' = \text{expression involving } x \text{ and } y$$