Handout: Linearizations, Linear Approximations, and Differentials

Definition of the Linearization

Words: The ***linearization*** of $f(x)$

Meaning: The function $L(x)$ defined by the equation

$$L(x) = f(a) + f'(a)(x - a)$$

Graphical Significance: $L(x)$ describes the line that is tangent to the graph of $f(x)$ at $x = a$.

Method for Finding a Linear Approximation

Given: a function $f(x)$ and a hard x value called $\hat{x}$. (That is, it is not easy, or maybe even not possible, to compute $f(\hat{x})$ exactly by hand.)

Goal: Find an ***approximation*** for $f(\hat{x})$.

Steps:

- Identify the function $f(x)$
- Identify the hard x value, called $\hat{x}$.
- Identify an easy nearby x value, called a. That is, such that $f(a)$ is easy to compute.
- Build the ***linearization*** of $f(x)$ at a. That is, build the function

$$L(x) = f(a) + f'(a)(x - a)$$

- Use the ***linearization*** to compute the number $L(\hat{x})$. That is, compute

$$L(\hat{x}) = f(a) + f'(a)(\hat{x} - a)$$

  (This should be an easy calculation.) This number $L(\hat{x})$ is the desired ***approximation*** for $f(\hat{x})$. It is called the ***linear approximation*** for $f(\hat{x})$.

Differentials

Consider function f and two known x values x_1 and x_2. How does the value of $y = f(x)$ change when x changes from x_1 to x_2?

- change in x: $\Delta x = x_2 - x_1 =$ change in x
- exact change in y: $\Delta y = f(x_2) - f(x_1)$ (conceptually simple but may be hard to comptue!)
- approximate change in y: $dy = f'(x_1) \cdot \Delta x$ (the differential of f at x_1)(easier to compute)