Rates of Change and Secant and Tangent Lines (Concepts from Section 2.1)

Definition of Average Rate of Change

Words: Average Rate of Change of f from a to b

Usage: a, b are real numbers, $a < b$, and f is a function that is continuous on the interval $[a, b]$.

Meaning: the number $m = \frac{f(b)-f(a)}{b-1}$

Graphical Significance: the number m is the slope of secant line that passes through points $(a, f(a))$ and $(b, f(b))$

Additional terminology: When the variable is t, representing *time* and the function $f(t)$ is a *position function*, representing the *position* of an object at time t, then the average rate of change is called the *average velocity* from time a to time b.

Alternate presentation of average rate of change:

Words: Average Rate of Change of f from a to $a + h$

Usage: a, h are real numbers, $h \neq 0$, and f is a function that is continuous on an interval near a

Meaning: the number $m = \frac{f(a+h)-f(a)}{h}$

Graphical Significance: the number m is the slope of secant line that passes through points $(a, f(a))$ and $(a + h, f(a + h))$

Definition of Instantaneous of Change

Words: Instantaneous Rate of Change of f at a

Symbol: $f'(a)$

Spoken: The derivative of f at a

Usage: a is a real number and f is a function that is continuous near $x = a$

Meaning: the number $m = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Additional terminology: When the variable is t, representing *time* and the function $f(t)$ is a *position function*, representing the *position* of an object at time t, then the Instantaneous rate of change is called the *instantaneous velocity* at time a

Definition of line tangent to graph of f at $x = a$

The line that has these two properties

- contains the *point* $(a, f(a))$ (This point is called the *point of tangency*.)
- has *slope* $m = f'(a)$ (This number is called the *slope of the tangent line* at $x = a$, but it is also called the *slope of the graph* of $f(x)$ at $x = a$.)

General Point Slope Form of the Equation of the Tangent Line

The line tangent to the graph of $f(x)$ at $x = a$ has equation

$$(y - f(a)) = f'(a)(x - a)$$