

## The Method of Integration by Substitution

Remember that the *Chain Rule for Derivatives* is used for taking the *derivative* of *nested functions*:

$$\text{Chain Rule for Derivatives: } \frac{d}{dx} \text{outer}(\text{inner}(x)) = \text{outer}'(\text{inner}(x)) \cdot \text{inner}'(x)$$

The goal now is to find the *general antiderivative* of a function  $f(x)$  that involves a *nested function*.

That is, we wish to find the *indefinite integral*  $\int f(x) dx$  where the integrand  $f(x)$  involves a nested function. This is not always possible. But sometimes it is, using the *Substitution Method*.

**The Substitution Method** for finding the *indefinite integral*  $F(x) = \int f(x) dx$

where the integrand  $f(x)$  involves a *nested function*.

**Step 1 Identify the inner function and call it  $u$ .** Write the equation  $\text{inner}(x) = u$  to introduce the single letter  $u$  to represent the inner function. Circle the equation.

**Step 2 Build the equation  $dx = \frac{1}{u'} du$ .** To do this, first find  $u'$ , then use it to build equation  $dx = \frac{1}{u'} du$ . Circle the equation.

**Step 3 Substitute, Cancel, Simplify.** In steps (1) and (2) you have two circled equations. **Substitute** these into the integrand of your indefinite integral. **Cancel** as much as possible and **simplify** by using the *Constant Multiple Rule*. The result should be a new basic integral involving just the variable  $u$ . (See **Remarks about Step 3** below.)

**Step 4 Integrate.** Find the new indefinite integral by using the indefinite integral rules. The result should be a *function form* involving just the variable  $u$  and  $+C$ .

**Step 5 Substitute Back.** Substitute  $u = \text{inner}(x)$  into your function from Step (4) The result will be a new function form involving just the variable  $x$  and the  $+C$ . This is the  $F(x)$  that we seek. Present the result clearly as  $F(x) = \text{BLAH}$  and circle it.

**Remarks about Step 3:** The result of **Step 3** should be a new indefinite integral with an integrand that is a function involving the variable  $u$ . There are three important things to check at the end of **Step 3**:

- There should be no  $x$  in the new indefinite integral. It should involve only  $u$ .
- The new indefinite integral should *not* involve a *nested function*, and it should be a *basic integral* that can be integrated using our indefinite integral rules.
- If the above two items are not satisfied, then either you made a mistake, or the original integral might be one for which the Substitution Method cannot be used.