MATH 2301 Handout for Wednesday March 20
Two Theorems from Section 4.2

Rolles Theorem: If function $f(x)$ satisfies these criteria (the ***hypotheses***)

- $f(x)$ continuous on [a,b]
- $f(x)$ differentiable on (a,b)
- $f(a) = f(b)$

Then the following statement is true (the **conclusion**)

There is a number c with $a < c < b$ such that $f'(c) = 0$.

In other words,

There is an $x = c$ with $a < c < b$ where the ***tangent line is horizontal***.

Remark: Theorem does not give you the value of c. If a c exists, you have to figure out its value.

The Mean Value Theorem: If a function $f(x)$ satisfies the following two requirements (the ***hypotheses***)

- f is *continuous* on the *closed interval* $[a, b]$
- f is *differentiable* on the *open interval* (a, b)

then the following statement (the ***conclusion***) is true:

There is a number $x = c$ (at least one) with $a < c < b$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

In other words,

The slope of the *tangent line* at c equals the slope of the *secant line* from a to b.

Remark: Theorem does not give you the value of c. If a c exists, you have to figure out its value.

Class Drill (exercise 4.2#13)

Consider the function $f(x) = \sqrt{x}$ on interval [0,4]

(a) Show that it satisfies the hypotheses of the Mean Value Theorem.

Is $f(x)$ continuous on the closed interval [0,4]? (Explain how you know.)

Is $f(x)$ differentiable on the open interval (0,4)? That is, does $f'(x)$ exist on the interval (0,4)? (To answer this question, you'll have to find $f'(x)$.)

(b) Find the value of c that works. Show the process.

Compute the value of $\frac{f(b)-f(a)}{b-a}$. This will be a number.

Use your formula for $f'(x)$ from part **(a)** to build the expression $f'(c)$. This will be an expression involving the variable c.

Set $f'(c) = \frac{f(b)-f(a)}{b-a}$ and solve for c.

(c) Illustrate the result on a graph of $f(x)$

Graph $f(x) = \sqrt{x}$ on interval $[0,4]$

Draw the secant line that touches the graph at $x = 0$ and $x = 4$

Draw the tangent line at $x = 1$

The two lines should look parallel. Label the lines with their slope m.