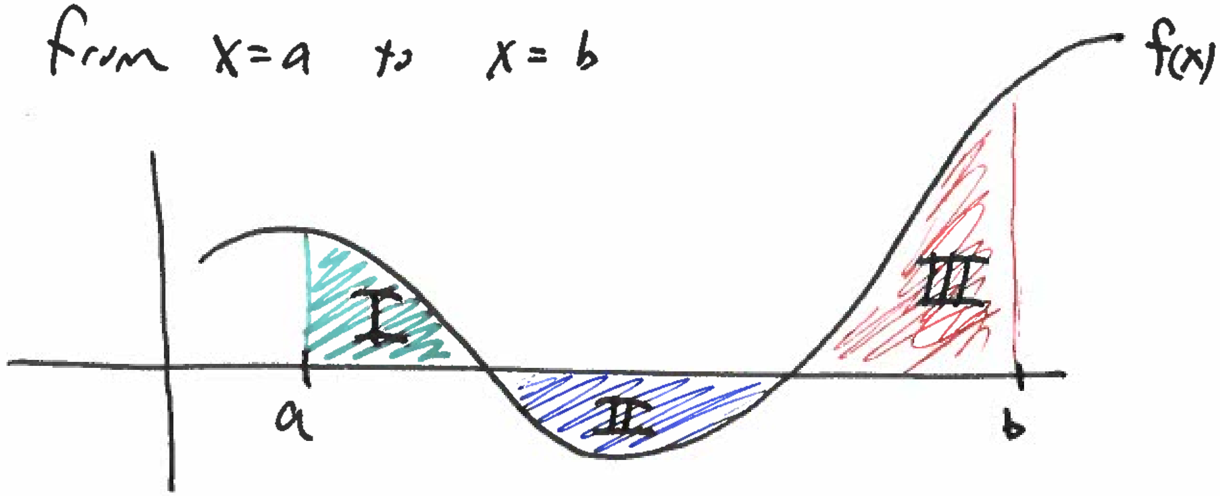
Monday April 8 Section 5.1 Area and Distance

Meeting Part 1 The Area Problem

Signed Area and Unsigned Area between graph of f(x) and x axis from x = a to x = b



unsigned area USA = I + II + III

Signed area SA = I − II + III

regions below the x axis get a negative sign.

In your mathematical life so far, you have learned how to compute area of basic geometric shapes using the appropriate formula

<u>The area question</u> (<u>The Area Problem</u>)

For a region that is not made up of basic geometric shapes

- what do we even <u>mean</u> by the "area"?
- how do we compute its value?

<u>Goal</u>: Answer the Area problem

Area, whatever it means, should behave this way:

positivity: If a region contains a disk  then the area ~~sh~~ must be >0.

additivity: If Region A $\subset$ Region B $\subset$ Region C then

$$\text{Area } A \leq \text{Area } B \leq \text{Area } C$$

Alternately: If Region I, Region II dont overlap, theen

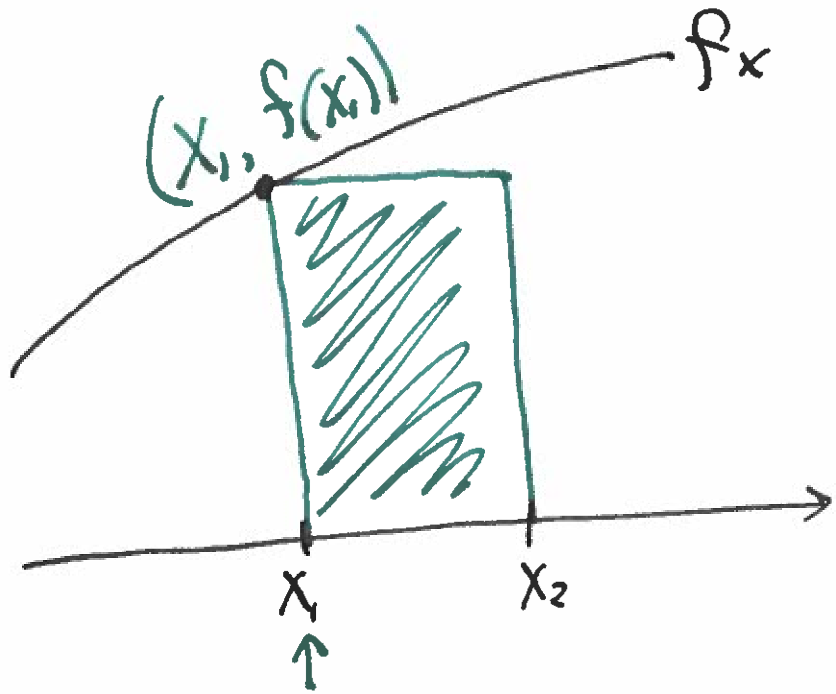
$$\text{Area}(\text{I} \cup \text{II}) = \text{Area}(\text{I}) + \text{Area}(\text{II})$$

Approximating Area of regions

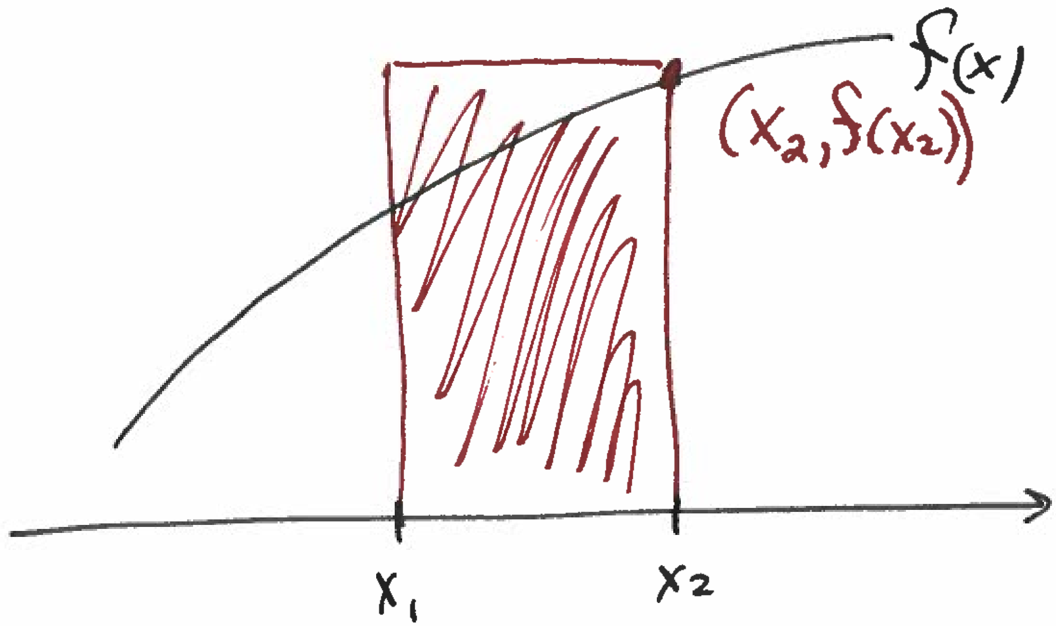
for a function $f(x)$ on an interval $[x_1, x_2]$

a left rectangle is a rectangle that

- sits on interval $[x_1, x_2]$
- has height $f(x_1)$. So the left corner of the rectangle is a point on the graph



A right rectangle on same interval has height $f(x_2)$ (4)
So it touches graph at rectangles right corner.



Riemann Sums

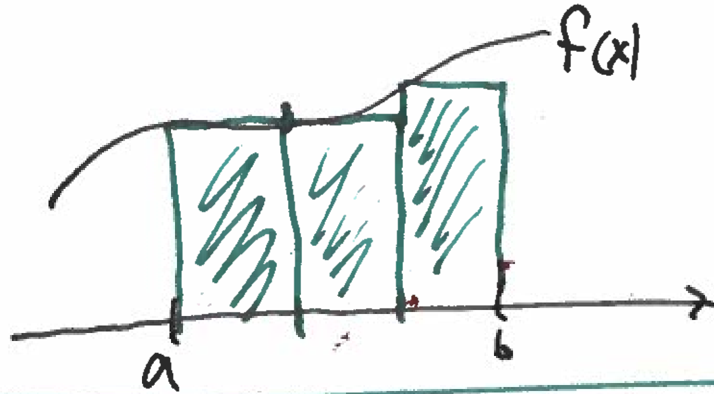
Define Left Riemann Sum

Symbol: L_n

Spoken: The left Riemann Sum with n rectangles.

Usage: A function f(x) is given and an interval [a, b] and f is continuous on the interval [a, b] or may have a finite number of discontinuities.

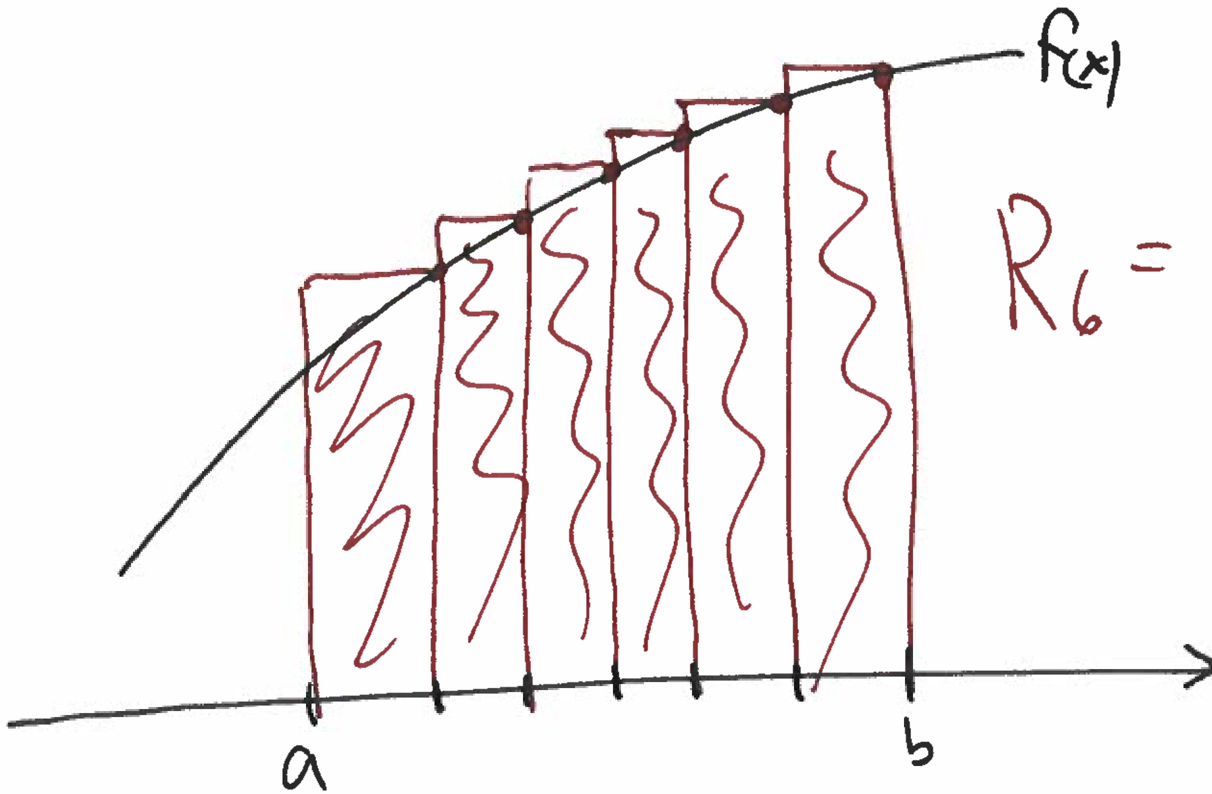
Meaning: L_n is the number that is the sum of the areas of n equal width left rectangles parked on the interval [a, b]



L_3 = Sum of those three areas

The Right Riemann Sum R_n

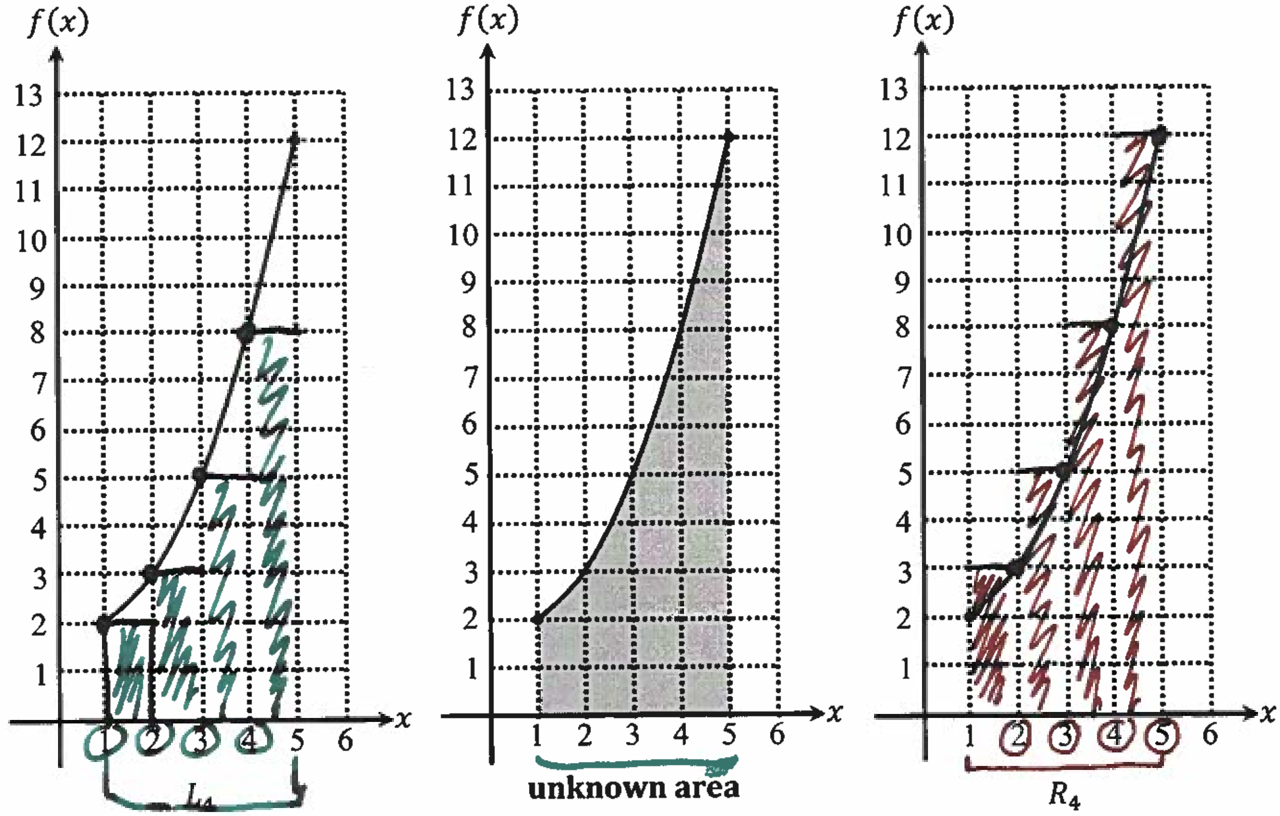
is the same thing, but using right rectangles



R_6 = Sum of these six areas

Class Drill: Estimating the Area Under a Graph by Using Riemann Sums

The goal is to estimate the shaded area in the middle figure. You will do this by finding the values of the Riemann sums L_4 and R_4. This will give you lower and upper bounds for the shaded area.



(A) Draw in the rectangles for the left sum L_4.

(B) Find the value of L_4. $L_4 = 2+3+5+8 = 18$

(C) Draw in the rectangles for the right sum R_4.

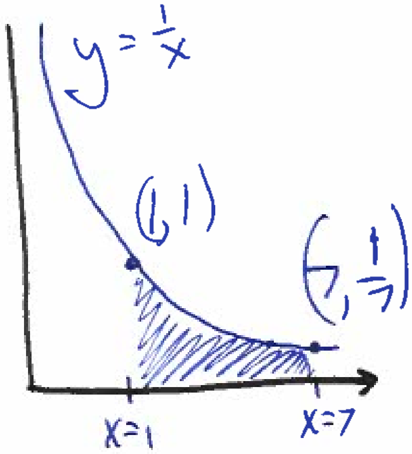
(D) Find the value of R_4. $R_4 = 3+5+8+12 = 28$

(E) Use the values from questions (B)and (D) to build a true inequality

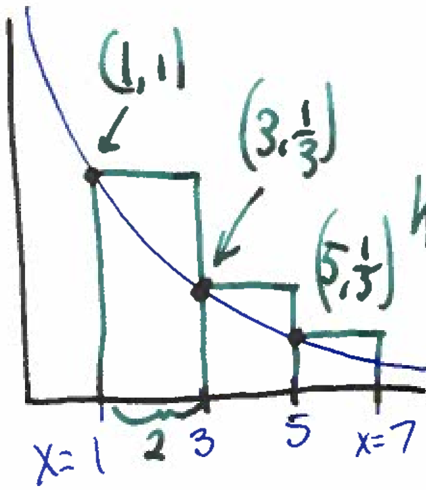
18 < **unknown area** < 28

Class Drill: Computing Riemann Sums

The goal is to find approximations for the signed area between the graph of the function $f(x) = \frac{1}{x}$ and the x axis on the interval [1,7] by computing Left and Right Riemann Sums with 3 rectangles. That is, find values for L_3 and R_3. Show all details clearly. (Hand calculations! No calculators or cell phones!)



$L_3 =$



widths all $\Delta x = \frac{7-1}{3} = 2$

heights:

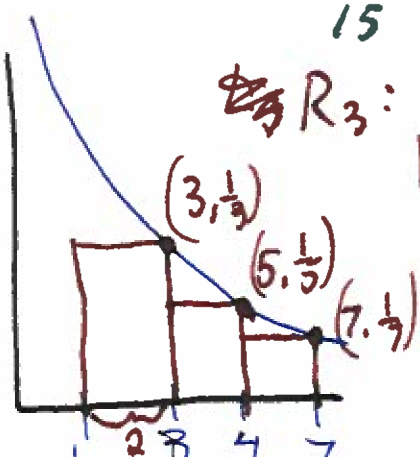
$f(1) = \frac{1}{1} = 1$

$f(3) = \frac{1}{3}$

$f(5) = \frac{1}{5}$

$= L_3 = 2 \cdot 1 + 2 \cdot \frac{1}{3} + 2 \cdot \frac{1}{5} = \left(1 + \frac{1}{3} + \frac{1}{5}\right) \cdot 2 = \frac{46}{15}$

$= \frac{46}{15}$



R_3: widths $\Delta x = 2$

heights $f(3) = \frac{1}{3}$ $f(5) = \frac{1}{5}$ $f(7) = \frac{1}{7}$

$R_3 = 2 \cdot \frac{1}{3} + 2 \cdot \frac{1}{5} + 2 \cdot \frac{1}{7} =$

$= \left(\frac{1}{3} + \frac{1}{5} + \frac{1}{7}\right) 2 = \left(\frac{35 + 21 + 15}{105}\right) 2 = \frac{142}{105}$