College Geometry Lecture #31 Mon April 8

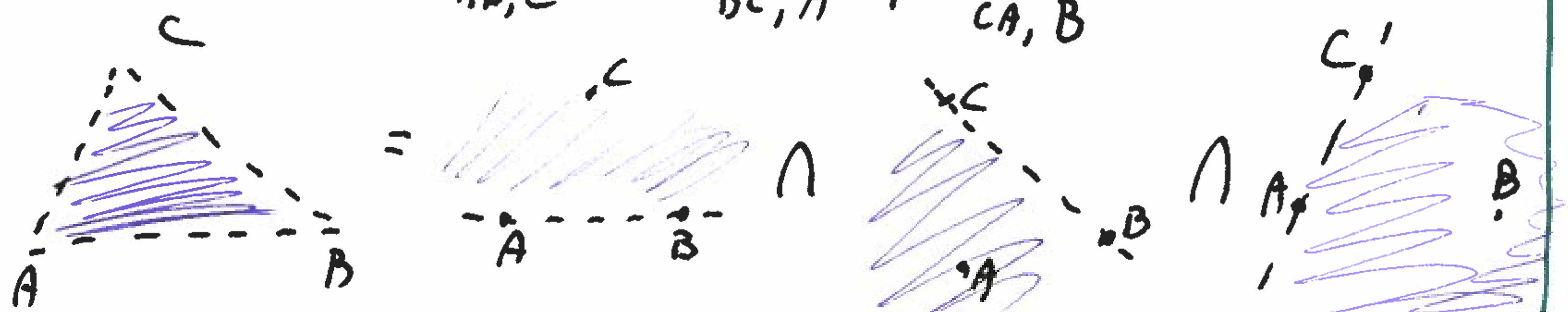
Meeting Part I Section 7.1 The Neutral Area Postulate

Definition 7.1.1 Interior of a triangle

<u>Symbol</u>: $\mathrm{int}(\triangle ABC)$

<u>Meaning</u>: The set of points

$$\mathrm{int}(\triangle ABC) = H_{\overleftrightarrow{AB},C} \cap H_{\overleftrightarrow{BC},A} \cap H_{\overleftrightarrow{CA},B}$$



Definition ~~7.1.2~~ 7.3.2 Triangular Region

$$▲ABC = \triangle ABC \cup \mathrm{int}(\triangle ABC)$$

Definition 7.1.3 Polygonal Region
is a set of points that is a union of triangular regions and that can be expressed as a union of triangular regions that only intersect along their edges

Example

Definition 7.1.4 Triangulation of a Polygonal Region is a particular way of expressing it as a union of triangular regions that only intersect along their edges

Definition 7.1.5 Non-overlapping polygonal regions are regions that only intersect along their edges.

Axiom 7.1.6 Neutral Area Axiom

There exists a function, α, (alpha) called the area function on the set of polygonal regions

$$\alpha : \underbrace{\text{the set of all polygonal regions}}_{\text{the domain}} \longrightarrow \underbrace{\mathbb{R}^+}_{\text{the codomain}}$$

positive real numbers >0

Such that

congruence: If ~~??~~ $\triangle ABC \cong \triangle DEF$, then $\alpha(\blacktriangle ABC) = \alpha(\blacktriangle DEF)$

additivity: If R_1, R_2 are non-overlapping polygonal regions,

then $\alpha(R_1 \cup R_2) = \alpha(R_1) + \alpha(R_2)$

Book 2nd Second Edition (your book) says $\underset{\geq 0}{\mathbb{R}^{nonneg}}$ instead of $\underset{>0}{\mathbb{R}^+}$

Definition 7.1.9 Polygonal Region determined by a quadrilateral.

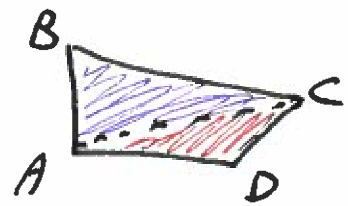
Symbol $\blacksquare ABCD$

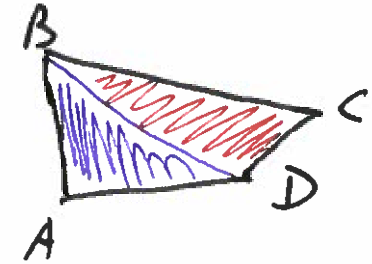
Usage: can be used when $\square ABCD$ is a quadrilateral

Meaning

Case 1 When $\square ABCD$ is convex, then

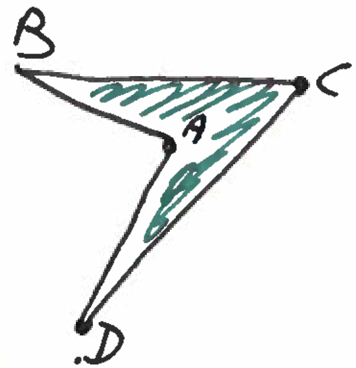
$\blacksquare ABCD$ means $\blacktriangle ABC \cup \blacktriangle ACD = \blacktriangle ABD \cup \blacktriangle CBD$



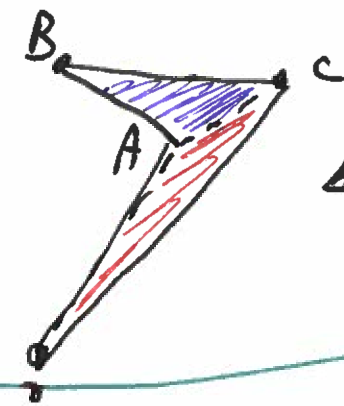


Case 2 when $\square ABCD$ is not convex with vertex A in interior($\angle BCD$) but vertex C not in interior of $\angle BAD$

$\blacksquare ABCD$



means



$\blacktriangle ABC \cup \blacktriangle ACD$

Meeting Part 2 Section 7.2 The Euclidean Area Axiom

Axiom 7.2.1 The Euclidean Area Axiom

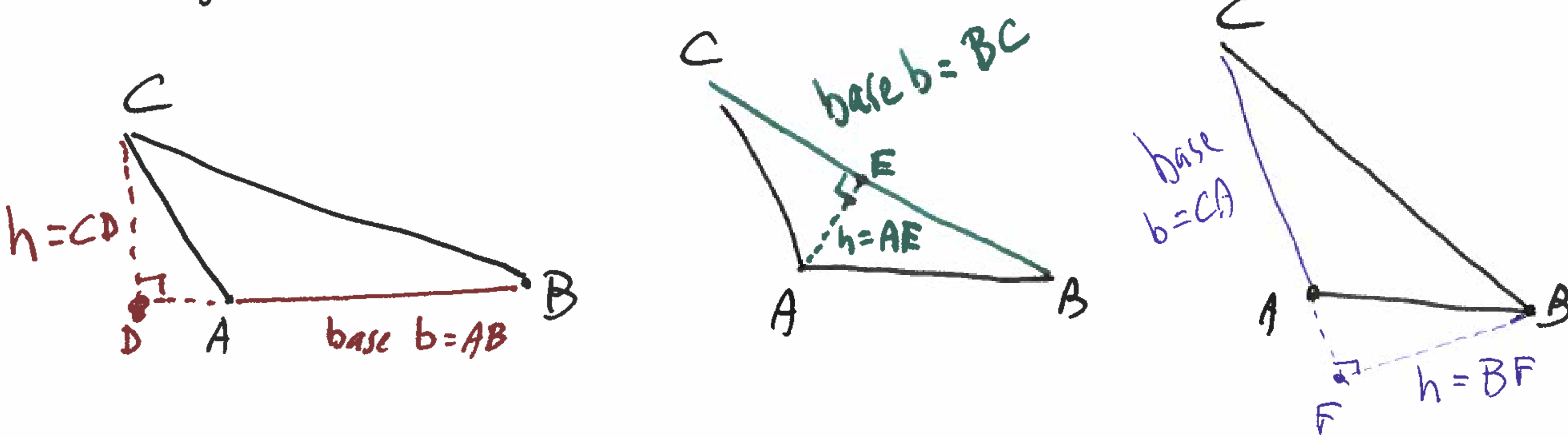
The area of a rectangular region bounded by a rectangle $\square ABCD$ is $\alpha(\blacksquare ABCD) = AB \cdot CD$

Terminology of base & height for triangle

A base for a triangle is simply a choice of one of the sides.
We sometimes use the word "base" for the length of that chosen side. And we use the letter b for that length.

Given a choice of base, with length b,
the corresponding height, h, is the length of the altitude segment from the opposite vertex

For any $\triangle ABC$, there are 3 possible bases, each with its own height.



Big Fact (Proven in exercise 7.2#3)

In any triangle, the product $b \cdot h$ does not depend on which side is used as the base.

Theorem 7.2.3 In Euclidean Geometry $\alpha(\triangle ABC) = \frac{1}{2}\,\text{base}\cdot\text{height}$ (which does not depend on choice of base.)

$\boxed{\text{End of Lecture}}$