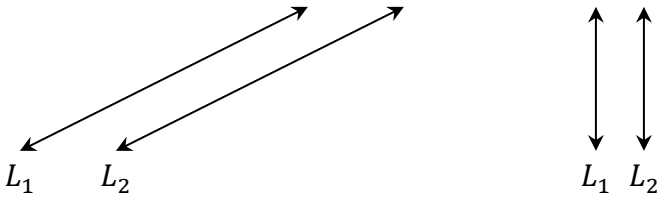
Handout for MATH 1300 (Barsamian) Lecture L10 Thu Sep 10: Parallel, Perpendicular

Definition of Parallel Lines

Words: Lines L_1 and L_2 are parallel
Symbol: $L_1 \parallel L_2$
Meaning: L_1, L_2 don't intersect. That is, there are no points that L_1, L_2 have in common.

Sample Parallel Lines:



Big Fact about Parallel Lines

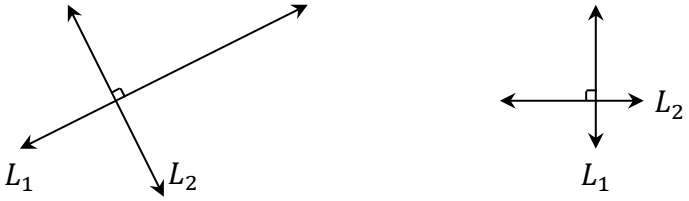
Lines L_1, L_2 can only be parallel when one of these conditions is met

- Lines L_1, L_2 have the same slope and are not the same line. (Observe that for this to be true, neither line can be vertical, because vertical lines have undefined slope.)
- Lines L_1, L_2 are both vertical and are not the same line. (Observe that for this to be true, the two lines both have undefined slopes, so one cannot say that they have the same slope.)

Definition of Perpendicular Lines

Words: Lines L_1 and L_2 are perpendicular
Symbol: $L_1 \perp L_2$
Meaning: L_1, L_2 intersect at a right angle.

Sample Perpendicular Lines:



Big Fact about Perpendicular Lines

Lines L_1, L_2 can only be perpendicular when one of these conditions is met

- Lines L_1, L_2 have slopes satisfying the equation

$$m_1 \cdot m_2 = -1$$

Remark: Observe that for this equation to be true, both m_1 and m_2 have to be real numbers (which tells us that neither of the lines can be vertical) and neither of the numbers can be zero (which tells us that neither of the lines can be horizontal). Since neither of those numbers is zero, we can divide both sides of the equation by either of those numbers to get a new equation. For instance $m_1 = -\frac{1}{m_2}$ or $m_2 = -\frac{1}{m_1}$ So the slopes are the negative reciprocals of each other.

- One of the lines is horizontal, the other is vertical.
Remark: Observe that in this case, one slope is zero and the other slope is undefined. So in this case, the equations $m_1 \cdot m_2 = -1$ and $m_1 = -\frac{1}{m_2}$ and $m_2 = -\frac{1}{m_1}$ are NOT true.

Drills for MATH 1300 (Barsamian) Lecture L10 Thu Sep 10: Parallel, Perpendicular

Sit in groups of two students. Groups will be called to work at the board.

No technology. (No calculators, computers, phones, tablets, watches)

[1] Consider the line $y = \frac{4}{3}x$.

(a) What is the slope of a line parallel to this line?

(b) What is the slope of a line perpendicular to this line?

[2] Consider the line $4x + 2y = 5$. Answer the same questions **(a)**, **(b)** as in **[1]**.

[3] The equations of three lines are given below.

- Line 1: $y = -\frac{2}{5}x + 4$
- Line 2: $y = -\frac{2}{5}x + 7$
- Line 3: $2x + 5y = 20$
- Line 4: $y = \frac{2}{5}x + 7$
- Line 5: $y = \frac{5}{2}x + 7$

For each pair of lines, determine whether they are parallel, perpendicular, or neither.

- Line 1 and Line 2: Parallel Perpendicular Neither
- Line 1 and Line 3: Parallel Perpendicular Neither
- Line 1 and Line 4: Parallel Perpendicular Neither
- Line 1 and Line 5: Parallel Perpendicular Neither
- Line 2 and Line 3: Parallel Perpendicular Neither
- Line 2 and Line 4: Parallel Perpendicular Neither
- Line 2 and Line 5: Parallel Perpendicular Neither
- Line 3 and Line 4: Parallel Perpendicular Neither
- Line 3 and Line 5: Parallel Perpendicular Neither
- Line 4 and Line 5: Parallel Perpendicular Neither

[4] Is the line that contains $(2,4)$ and $(4,5)$ parallel to the line that contains $(6,6)$ and $(10,8)$?

[5] Line L is given by the equation $y = 2x + 3$

(a) Find an equation for the line that is perpendicular to L and contains the point $(4,5)$.

(b) Find an equation for the line that is parallel to L and contains the point $(4,5)$.

(c) Draw the given line and the lines from **(a)**, **(b)**.

[6] Line L is given by the equation $x = 7$. Answer the same questions **(a)**, **(b)**, **(c)** as in **[5]**.

[7] Line L is given by the equation $y = 7$. Answer the same questions **(a)**, **(b)**, **(c)** as in **[5]**.