Handout for MATH 1300 (Barsamian) Lecture L17 Th Sep 24 Section 2.2

Notation for Trends

This notation becomes very important in Calculus

Symbol: $x \to \infty$

Spoken: x goes to infinity

Meaning: The values of x become positive without bound

Significance for a graph: Refers to the right end of the graph

Obvious generalization: $x \to -\infty$

Symbol: $f(x) \to \infty$

Spoken: $f(x)$ goes to infinity

Meaning: The values of $f(x)$ become positive without bound

Significance for a graph: Refers to the y values on a graph of $f(x)$

Obvious generalization: $f(x) \to -\infty$

Roots (zeros) of functions

Words: ***root*** of a function, or ***zero*** of a function $f(x)$

Meaning: An x that causes a y value of zero. That is, a number c such that $f(c) = 0$.

Graphical Significance: The graph of $f(x)$ has an x intercept at the point $(c, 0)$.

Roots (zeros) of polynomial functions

Huge Fact: c is a root (a zero) of polynomial $f(x) \Leftrightarrow (x - c)$ is a factor of $f(x)$

Terminology: The ***multiplicity*** of a root c is the ***power*** m of the factor $(x - c)$ in the factorization of $f(x)$.

Fact:

- If c is a zero of $f(x)$ with odd multiplicity, then the graph crosses the x axis at $(c, 0)$. That point is called a ***cross point***.
- If c is a zero of $f(x)$ with even multiplicity, then the graph touches the x axis at $(c, 0)$ but does not cross. That point is called a ***touch point***.

[1] For each function, determine whether it is a polynomial function. If it is, give its degree, leading coefficient, and constant term. (If the function is not a polynomial, skip the other questions.)

	function	polynomial?	degree	leading coefficient	constant term
1	$y = 6x - 7x^2$				
2	$y = 0x^5 + 6x - 7x^2 + 13$				
3	$y = 6x - \frac{7}{x^2}$				
4	$y = 5x^4 + x^{2/3} - 6$				
5	$y = 5x^4 + \frac{2}{3}x^7 - 6$				
6	$y = 5x^4 + \frac{2}{3}x^{\pi} - 6$				
7	$y = -7 + 5\sqrt{x}$				
8	$y = 5$				
9	$y = 0$				

[2] Find all zeros of the quadratic function $y = x^2 + 5x - 14$. What are their multiplicities?

[3] For the function

$$f(x) = (x+8)^2(x+7)^3(x-11)(x-13)$$

(a) Is the function a polynomial?
(b) What is the degree?
(c) What is the leading coefficeint?
(d) What is the end behavior of the graph?
(e) List each zero of f and give its multiplicity. Is the multiplicity even or odd?
(f) What is the y intercept of the graph? (Just give an expression for the number, not the actual number.
(g) Sketch the graph.

[4] Write a formula for a polynomial function whose graph goes down on the left and down on the right and has the following other properties: Hint: One of these is impossible. Which one? Why?
(a) touch point at $x = -2$ and touch point at $x = 3$.

(b) cross point at $x = -2$ and cross point at $x = 3$.

(b) cross point at $x = -2$ and touch point at $x = 3$.