Handout for MATH 1300 (Barsamian) Lecture L18 Mon Sep 28, 2026 Sections 2.3, 2.4

Definition of Polynomial

A ***non-zero polynomial*** is an expression of the form

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + +a_1 x + a_0$$

Where n is an integer with $n \geq 0$ and the $a_0, a_1, \ldots, a_{n-1}, a_n$ are real numbers called coefficients, with $a_n \neq 0$. The real number a_n is called the ***leading coefficient***, and the integer n is called the ***degree*** of the polynomial.

The ***zero polynomial*** is just the real number 0. The zero polynomial does not fit the form presented above. The zero polynomial has undefined degree and undefined leading coefficient.

Division Algorithm: Suppose that $f(x)$ and $d(x)$ are polynomials with $d(x) \neq 0$ and the degree of $d(x)$ is less than or equal to the degree of $f(x)$. Then there exist unique polynomials $q(x)$ and $r(x)$ such that.

$$f(x) = d(x) \cdot q(x) + r(x)$$

where the degree of $r(x)$ is less than the degree of $d(x)$, or $r(x)$ is the zero polynomial.

Note: In the equation above, the polynomials are given the following names:

$$Dividend(x) = Divisor(x) \cdot Quotient(x) + Remainder(x)$$

Synthetic Division

Project Method from Book

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

The Factor Theorem Theorem

c is a root (a zero) of polynomial $f(x) \Leftrightarrow (x - c)$ is a factor of $f(x)$

The Rational Zero Theorem

If polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + +a_1 x + a_0$ has integer coefficients, and if $\frac{p}{q}$ (written in lowest terms) is a rational zero of f (that is, a root of f), then

- p is a factor of the constant term a_0
- q is a factor of the leading coefficient a_0

[1] Find

$$\frac{8x^2 + 34x + 35}{2x + 5}$$

using long division. Write the result in two ways

- In the form

$$\frac{Dividend}{Divisor} = Quotient$$

- In the form

$$Dividend = Divisor \cdot Quotient$$

[2] Find

$$\frac{3x^3 - 10x^2 + 13x - 17}{x - 2}$$

using long division. Write the result in two ways

- In the form

$$\frac{Dividend}{Divisor} = Quotient + \frac{Remainder}{Divisor}$$

- In the form

$$Dividend = Divisor \cdot Quotient + Remainder$$

[3] Use Synthetic Division to find

$$\frac{3x^3 - 10x^2 + 13x - 17}{x - 2}$$

[4] The goal is to find $P(2)$ for the polynomial

$$P(x) = 5x^3 - 11x^2 + 3x + 1$$

(a) Do the associated division problem. Write the result in two ways

- In the form

$$\frac{Dividend}{Divisor} = Quotient + \frac{Remainder}{Divisor}$$

- In the form

$$Dividend = Divisor \cdot Quotient + Remainder$$

(b) Now use the remainder theorem to find $P(2)$.

[5] Use the Factor Theorem to determine whether $x - 3$ is a factor of the polynomial

$$P(x) = 2x^4 - 5x^3 + x - 30$$

Specifically, evaluate P at the proper value, and then determine whether $x - 3$ is a factor.

[6] Use the Rational Zeros theorem to find all zeros of

$$f(x) = 2x^3 + 7x^2 + 2\,x - 3$$

[7] Use the Rational Zeros theorem to find all zeros of

$$f(x) = 5x^4 - 44x^3 + 60x^2 + 23x - 14$$