Handout for MATH 1300 (Barsamian) Lecture L19 Wed Sep 30, 2026 Section 2.5

Definition of Rational Function

A ***rational function*** is a ratio of polynomials where the denominator is not the zero polynomial.

$$f(x) = \frac{p(x)}{q(x)}$$

The domain of a rational function is all real numbers except the numbers c that are zeroes of the denominator $q(x)$.

Identifying Vertical Asymptotes of a Rational Function

For a rational function

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ have no common factors, if c is a zero of the denominator $q(x)$, then the graph of f has a vertical asymptote with line equation $x = c$.

Identifying Horizontal Asymptotes of a Rational Function

For a rational function

$$f(x) = \frac{p(x)}{q(x)}$$

where the numerator $p(x)$ has degree n and leading coefficient a, and the denominator $q(x)$ has degree m and leading coefficient b.

If $n = m$, then the graph of f has a horizontal asymptote with line equation $y = \frac{a}{b}$.

If $n < m$, then the graph of f has a horizontal asymptote with line equation $y = 0$.

If $n > m$, then the graph of f does not have a horizontal asymptote.

Identifying Slant and Nonlinear Asymptotes of a Rational Function

For a rational function

$$f(x) = \frac{p(x)}{q(x)}$$

where the numerator $p(x)$ has degree n and leading coefficient a, and the denominator $q(x)$ has degree m and leading coefficient b.

- If $n = m + 1$, then the graph of f has a slant asymptote asymptote. To find the equation of the slant asymptote, use long division to compute

  $$\frac{p(x)}{q(x)}$$

  Write the result in the form

  $$\frac{p(x)}{q(x)} = Quotient(x) + \frac{Remainder(x)}{q(x)}$$

  Then $Quotient(x)$ will be the equation will be the formula for the slant asymptote
- If $n > m + 1$, then the end behavior of the function resembles the end behavior of the power function $y = \frac{ax^n}{bx^m}$.

.

[1] Find the line equations for all vertical and horizontal asymptotes for the rational function

$$f(x) = \frac{7}{4x - 6}$$

and then draw the asymptotes. Label the asymptotes with their line equations.

[2] Find the line equations for all vertical and horizontal asymptotes for the rational function

$$f(x) = \frac{2x + 3}{x^2 + 2x - 3}$$

Also find the axis intercepts.
Then draw the asymptotes and label them with their line equations, and plot the axis intercepts and label them with their (x, y) coordinates.

[3] Same instructions as in [2], but for the function

$$f(x) = \frac{6x + 5}{3x - 3}$$

[4] Same instructions as in [2], but for the function

$$f(x) = \frac{3x^2 + 6x + 3}{-x^2 + x + 6}$$

[5] Find the line equations for all vertical, horizontal, or slant asymptotes for the rational function

$$f(x) = \frac{x^2 + 5x - 6}{x + 1}$$

Also find the axis intercepts.
Then draw the asymptotes and label them with their line equations, and plot the axis intercepts and label them with their (x, y) coordinates.

[6] Same instructions as in [5], but for the function

$$f(x) = \frac{4x^2 - 1}{2x + 3}$$