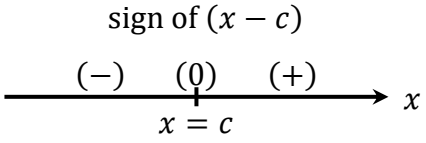
Handout for MATH 1300 (Barsamian) Lecture L20 Thu Oct 1, 2026 Sections 2.6, 2.7

Part 1: Sign Behavior of Functions (Section 2.7)

Consider sign behavior of the term $(x - c)$.

- If $x = c$, then $x - c = 0$
- If $x < c$, then $x - c < 0$
- If $x > c$, then $x - c > 0$

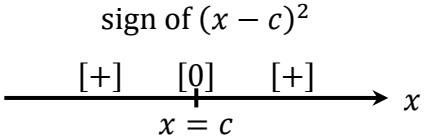
It is useful to display this on a number line, as shown



Now consider sign behavior of the term $(x - c)^2$.

- If $x = c$, then $(x - c)^2 = 0$
- If $x \neq c$, then $(x - c)^2 > 0$

This is displayed on a number line at right. Note the brackets.



The realization that factors change sign only at the x value that is the root of the factor can be used to determine the sign behavior of more complicated functions. If the function is put into factored form, then a number line can be used to keep track of the signs of all the various factors and the sign of the whole function. The number line with the sign analysis shown is my version of the ***sign chart***. Different books present sign charts in different ways. Many use tables; I prefer number lines.

[1] (a) Make a sign chart for $f(x) = (x + 3)(x - 1)(x - 6)$
(b) Solve the inequality $(x + 3)(x - 1)(x - 6) \leq 0$
(c) Make a sign chart for $f(x) = (x + 3)(x - 1)^2(x - 6)$
(d) Solve the inequality $(x + 3)(x - 1)^2(x - 6) \leq 0$

[2] (a) Make a sign chart for $g(x) = \frac{(x+3)(x-6)}{(x-1)}$
(b) Solve the inequality $\frac{(x+3)(x-6)}{(x-1)} \leq 0$

[3] The goal is to solve the inequality $x^3 + x^2 > 6x$
(a) Write an equivalent inequality that comparing a function $f(x)$ and the number 0. Write $f(x)$ in factored form.
(b) Make a sign chart for $f(x)$.
(c) Use your sign chart for $f(x)$ to solve the inequality.

[4] The goal is to solve the inequality $\frac{3x+4}{x-4} \geq x - 1$
(a) Write an equivalent inequality that comparing a function $g(x)$ and the number 0. Write $g(x)$ in factored form.
(b) Make a sign chart for $g(x)$.
(c) Use your sign chart for $g(x)$ to solve the inequality.

Part 2: Graphing Rational Functions by considering End Behavior, Asymptotes, Axis Intercepts, Sign Behavior (Section 2.6)

[5] The goal is to graph the rational function

$$f(x) = \frac{-x^2 - 7x}{x^2 + x - 12}$$

(a) Identify all the axis intercepts. Write their (x, y) coordinates.
(b) Identify all the asymptotes. Write their line equations.
(c) Make a sign chart for $f(x)$. Note that you'll need to put $f(x)$ into factored form first.
(d) With the information from (a),(b),(c), graph $f(x)$.

[6] The goal is to graph the rational function

$$f(x) = \frac{-x^2 + x + 12}{x + 2}$$

(a) Identify all the axis intercepts. Write their (x, y) coordinates.
(b) Identify all the asymptotes. Write their line equations. (Note that one asymptote will be a slant asymptote. To get the equation for the slant asymptote, you'll need to do long division to get the equation

$$f(x) = \frac{-x^2 + x + 12}{x + 2} = quotient + \frac{remainder}{x + 2}$$

The $quotient$ expression will be the formula for the slant asymptote.
(c) With the information from (a),(b), plot the axis intercepts and draw the asymptotes. Label them with their (x, y) coordinates and line equations.
(d) Make a sign chart for $f(x)$. Note that you'll need to put $f(x)$ into factored form first.
(e) Also consider the equation

$$f(x) = \frac{-x^2 + x + 12}{x + 2} = quotient + \frac{remainder}{x + 2}$$

The sign of the $\frac{remainder}{x+2}$ term will tell you when the graph of $f(x)$ lies above or below the slant asymptote.
(f) With the information from (a),(b),(c),(d),(e), graph $f(x)$.

[7] The goal is to graph the rational function

$$h(x) = \frac{x^2 - 2x - 3}{x - 3}$$

(a) Identify all the axis intercepts. Write their (x, y) coordinates.
(b) Identify all the horizontal or vertical asymptotes. Write their line equations.
(c) Identify all holes on the graph.
(d) With the information from (a),(b),(c), graph $h(x)$.