Handout for MATH 1300 (Barsamian) Lecture L24 Wed Oct 14, 2026

Meeting Part 1: Exponential Growth and Decay (Section 3.2)

Terminology of Exponential Growth and Decay

words: Quantity $Q(t)$ has exponential growth with relative growth rate r (expressed as a decimal)

meaning: $Q(t) = Q_0 e^{rt}$

words: Quantity $Q(t)$ has exponential decay with relative decay rate r (expressed as a decimal)

meaning: $Q(t) = Q_0 e^{-rt}$

[1] The mass of a radioactive substance follows a continuous exponential decay model. A sample of this radioactive substance has an initial mass of 425 kilograms and decreases continuously at a relative rate of 18% per day.

(a) Write a formula for the mass of the sample at time t days.

(b) Find the mass of the sample after two days. Give an exact answer in symbols, and also give a decimal approximation, rounded to 2 decimal places.

Meeting Part 2: Logarithmic Functions (Section 3.3)

Definition and basic properties

An exponential function is a function of the form $y = b^x$, with b satisfying $0 < b < 1$ or $b > 1$

Domain of b^x: all $x \in \mathbb{R}$

Range of b^x: all $y > 0$

Every exponential function is one-to-one so has an inverse function, which is denoted $y = \log_b(x)$.

Domain of $\log_b(x)$: all $x > 0$

Range of $\log_b(x)$: all $y \in \mathbb{R}$

Because exponential and logarithmic functions are inverses, we have the ***cancellation equations***:

$b^{\log_b(x)} = x$ for all $x > 0$

$\log_b b^x = x$ for all $x \in \mathbb{R}$

Two equations that express the same relationship between x and y:

$$\underbrace{y = \log_b(x)}_{\text{logarithmic form}} \Leftrightarrow \underbrace{x = b^y}_{\text{exponential form}}$$

We can exploit this equivalence to determine two particular simple values of logarithm functions.

$\log_b(1) = 0$ because $b^0 = 1$

$\log_b(b) = 1$ because $b^1 = b$

Additional terminology and notation

The ***natural exponential*** and ***natural logarithm*** functions have base $b = e$.

natural exponential: $y = e^x$

natural logarithm: $y = \log_e(x) \underset{\text{definition}}{=} \ln(x)$

The ***common exponential*** and ***common logarithm*** functions have base $b = 10$.

common exponential: $y = 10^x$

common logarithm: $y = \log_{10}(x) \underset{\text{definition}}{=} \log(x)$

Implications of the one-to-one property of exponential and logarithmic functions.

Since the exponential function b^x is one-to-one, we know that if $b^x = b^y$ then $x = y$.

Since the logarithmic function $\log_b x$ is one-to-one, we know that if $\log_b x = \log_b y$ then $x = y$.

[2] Rewriting Expressions

(a) Rewrite $5^{-1} = \frac{1}{5}$ as a logarithmic equation.

(b) Rewrite $\log_2 16 = 4$ as an exponential equation.

(c) Rewrite $e^5 = x$ as a logarithmic equation.

(d) Rewrite $\ln 2 = y$ as an exponential equation.

[3] Evaluate each expression

(a) $\log_6(36) =$

(b) $\log_4\left(\frac{1}{64}\right) =$

[4] The goal is to make a graph of $y = -\log_2(x-4) + 3$

(a) Graph $y = 2^x$. Label three points with their (x, y) coordinates label the asymptote with its line equation. What is the domain? what is the range?

(b) Graph $y = \log_2(x)$. Label three points with their (x, y) coordinates label the asymptote with its line equation. What is the domain? what is the range?

(c) Graph $y = -\log_2(x)$. Label three points with their (x, y) coordinates label the asymptote with its line equation. What is the domain? what is the range?

(d) Graph $y = -\log_2(x-4) + 3$. Label three points with their (x, y) coordinates label the asymptote with its line equation. What is the domain? what is the range?

Meeting Part 3: Properties of Exponents and Logarithms (Section 3.4)

Properties	for exponents	for logarithms
Product Property	$b^x b^y = b^{x+y}$	$\log_b(xy) = \log_b(x) + \log_b(y)$
Quotient Property	$\frac{b^x}{b^y} = b^{x-y}$	$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
Power Property	$(b^x)^p = b^{(x \cdot p)}$	$\log_b(x^p) = p\log_b(x)$

[5] Find the unknown value

(a) $\log_8(5) - \log_8(9) = \log_8(?)$

(b) $\log_2(?) + \log_2(11) = \log_2(99)$

(c) $2\log_7(5) = \log_7(?)$

[6] Write as a single logarithm.

$7\log_6(6z+1) + \frac{1}{5}\log_6(z+8) =$

[7] Use the properties of logarithms to expand into logarithm expressions that each involve only variable and that do not have any radicals or exponents. All variables are positive.

$\log\left(\frac{5\sqrt{x^3 y}}{z^7}\right) =$