Handout for MATH 1300 (Barsamian) Lecture L25 Thu Oct 15, 2026 Section 3.5 Exponential and Logarithmic Equations

Implications of the one-to-one property of exponential and logarithmic functions.

Since the exponential function b^x is one-to-one, we know that if $b^x = b^y$ then $x = y$.

Since the logarithmic function $\log_b x$ is one-to-one, we know that if $\log_b x = \log_b y$ then $x = y$.

[1] Solve for x by using the one-to-one property.

(a) $\log(8) = \log(4x - 2)$

(b) $16 = 8^{(-x+2)}$ **Hint:** First rewrite as an equivalent equation with the same base on both sides.

Solving Exponential Equations by Using Logarithms

1. Given an exponential equation, rewrite it in an equivalent form with a single exponential expression on one side of the equation, or with a single exponential expression on each side.
2. Take a logarithm of the same base on both sides of the equation.
3. Use the power property of logarithms to "bring down" the exponent.
4. Solve the resulting equation.

[2] Solve $14^{-7x} = 11^{x+7}$ for x. Write an exact answer using either base 10 or base e logarithms.

Solving Exponential Equations by Using Substitution and Quadratic Factoring

The equation

$$ae^{2x} + be^x + c = 0$$

cannot be solved using the method of logarithms described above. But notice that $e^{2x} = (e^x)^2$. So if we substitute $y = e^x$, the equation becomes

$$ay^2 + by + c = 0$$

This equation can be solved for y by standard methods of solving quadratic equations. Then x can be found by the computing $x = \ln y$.

[3] Solve $e^{2x} - 10e^x + 21 = 0$ for x.

Solving Logarithmic Equations by Using Exponential Form

1. Given a logarithmic equation, first note the domain of the equation.
2. Isolate the logarithms on one side of the equation.
3. Use properties of logarithms to write the equation in the form $\log_b x = k$, where k is a constant.
4. Write the equation in exponential form.
5. Solve the equation from step 4. Keep only the solutions that are in the domain.

[4] Solve for x. Give exact answers in symbols, and then give decimal approximations

(a) $8 + \log_5(-4x) = 10$.

(b) $7 - 3\ln x = 6$

(c) $\log_4(x - 7) = 1 + \log_4(x - 4)$