Handout for MATH 1300 (Barsamian) Lecture L28 Wed Oct 21, 2026 Section 4.1 Angles and their Measure

Definition of Radian Measure of a Central Angle

$$\text{measure of central angle in radians} = \frac{\text{length of the subtended arc}}{\text{radius of the circle}}$$

$$\theta = \frac{s}{r}$$

Converting between Degree and Radian Measure

$$\pi = 180°$$

$$\pi \text{ radians} = 180 \text{ degrees}$$

Dividing the above equation can give us two ratios that each equal 1

$$\left(\frac{\pi \text{ radians}}{180 \text{ degrees}}\right) = 1 \text{ and } \left(\frac{180 \text{ degrees}}{\pi \text{ radians}}\right) = 1$$

These ratios, which are just fancy ways of writing the number 1, can be used to convert units.

Example: To convert 37 degrees to radians, we can do the following calculation

$$37 \text{ degrees} \underset{\text{obvious}}{=} 37 \text{ degrees} \cdot 1 \underset{\text{trick}}{=} 37 \text{ degrees} \cdot \left(\frac{\pi \text{ radians}}{180 \text{ degrees}}\right) \underset{\text{cancel}}{=} \frac{37\pi}{180} \text{radians}$$

Being careful with the units insures that you don't do the wrong thing. Suppose that, in the above calculation, instead of multiplying by $\frac{\pi}{180}$, you had multiplied by $\frac{180}{\pi}$. If you brought the units along, you would discover that the units don't work out nicely. You would end up with a true equation, but inconvenient units, so you would know that you did the wrong thing.

$$37 \text{ degrees} = 37 \text{ degrees} \cdot \left(\frac{180 \text{ degrees}}{\pi \text{ radians}}\right) = \frac{37 \cdot 180}{\pi} \cdot \frac{\text{degrees}^2}{\text{radians}}$$

The resulting equation is true, but it is not helpful, because the units are not the desired units

[1] Give exact, simplified answers in symbols, not decimal approximations.

(a) Convert $-\frac{7\pi}{6}$ radians to degrees. (b) Convert 420° to radians

(c) Convert $\frac{8\pi}{11}$ radians to degrees. (d) Convert 191° to radians.

[2] Sketch the following angles in standard position. For each, sketch a coterminal angle that has value between 0° and 360° (or between 0 and 2π). Find the measure of the coterminal angle.

(a) $-240°$ (b) $-\frac{2\pi}{3}$ (c) $-225°$ (d) $-\frac{5\pi}{3}$ (e) 765° (f) $\frac{25\pi}{6}$ (g) 690° (h) $-\frac{11\pi}{3}$ (i) $-330°$ (j) $\frac{5\pi}{2}$

[3] (a) Draw a unit circle. On that circle, draw an arc that subtends a central angle that has angle measure $\theta = \frac{\pi}{2}$ radians. Find s, the length of the arc. Write your answer in terms of π.

(b) Same instructions, but with radius $r = 3$.

[4] (a) Draw a circle of radius $r = 2$. On that circle, draw an arc with arc length $s = \frac{2\pi}{3}$. Draw the corresponding central angle. Find the angle measure θ for that central angle.

(b) Same instructions, but with radius $r = 1$.