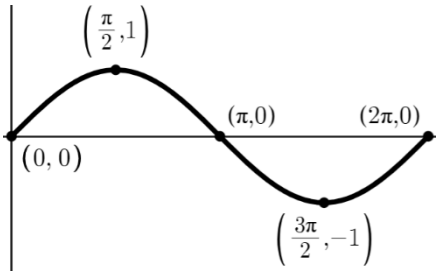
Handout for MATH 1300 (Barsamian) Lecture L32 Thu Oct 29, 2026

Section 4.5 Graphs of sin and cos Functions

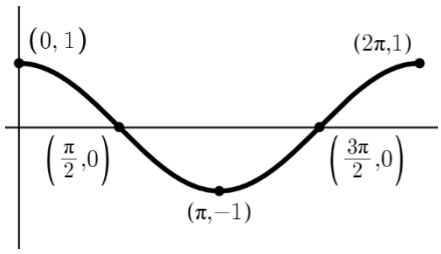
In Section 4.5, you will make graphs that are transformations of the basic sin and cos graphs.

It is important that you have memorized the five important points of one period of those basic graphs, both how those five points look on the graph and how they look in an input, output table.

x	$\sin(x)$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0



x	$\cos(x)$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1



Earlier in the semester, I presented a table that would assist in making transformations of a graph. In that table, one scrutinized the constants involved in a transformation, deciding whether they were additive or multiplicative constants, and whether they are on the inside or on the outside. This information told us about the transformation of the graph that is caused by each constant. For functions that involved numerous constants, this was a multi-step process and could be long.

There is another way of thinking about transformations of graphs, one where you consider a complicated composition of functions as a bunch of simple compositions, one after another. The compositions can be recorded as columns in a table of data that is then used to make a graph.

[Example 1] For instance, consider the function $y = 2\sin\left(x - \frac{\pi}{2}\right) + 3$. Think about what happens when you feed an input x into this function.

- Given input x,
- subtract $\frac{\pi}{2}$ from that input to get $x - \frac{\pi}{2}$,
- then take the sin of that result to get $\sin\left(x - \frac{\pi}{2}\right)$,
- then multiply that result by 2 to get $2\sin\left(x - \frac{\pi}{2}\right)$,
- then finally, add 3 to that result to get $2\sin\left(x - \frac{\pi}{2}\right) + 3$. This is the output.

This can be visualized with a table

(input) x	$x - \frac{\pi}{2}$	$\sin\left(x - \frac{\pi}{2}\right)$	$2\sin\left(x - \frac{\pi}{2}\right)$	$2\sin\left(x - \frac{\pi}{2}\right) + 3$ (the output)

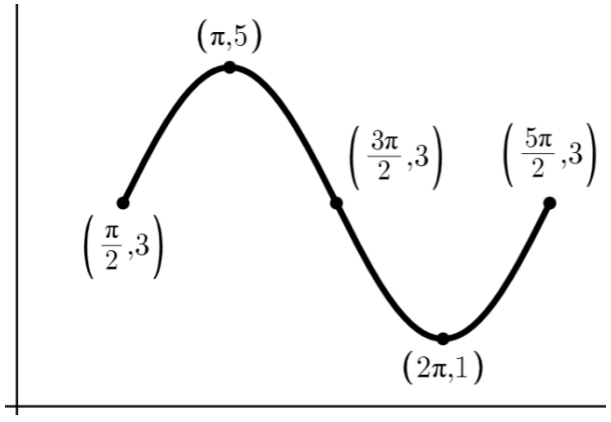
Now, consider what we know about the sin function. We have five famous points from the earlier table on page 1. That is, we know five convenient input/output pairs for the sin function. Those are data points that can be put into this table, giving us five rows that have two columns filled in. But realize that the two columns that are filled in are embedded inside the table. That is, the data goes in as input/output values for the sin function, wherever that occurs in the table.

(input) x	$x-\frac{\pi}{2}$	$\sin\left(x-\frac{\pi}{2}\right)$	$2\sin\left(x-\frac{\pi}{2}\right)$	$2\sin\left(x-\frac{\pi}{2}\right)+3$ (the output)
	0	0		
	$\frac{\pi}{2}$	1		
	π	0		
	$\frac{3\pi}{2}$	-1		
	2π	0		

Now consider what needs to be done to fill in the rest of the columns. To fill in the columns to the right, you just do the operations shown. That is, you multiply the numbers in column 3 by 2 to get the numbers for column 4. Then you add 3 to all the numbers in column 4 to get the numbers for column 5. But to fill in column 1, you have to work *backwards* from column 2. That is, you have to *add* $\frac{\pi}{2}$ to the values in column 2 to get the values for column 1. (You can see why: The numbers in column 1 are going to get $\frac{\pi}{2}$ *subtracted* from them to get the numbers in column 2, so it makes sense that one should *add* $\frac{\pi}{2}$ to the values in column 2 to get the values for column 1.)

(input) x	$x-\frac{\pi}{2}$	$\sin\left(x-\frac{\pi}{2}\right)$	$2\sin\left(x-\frac{\pi}{2}\right)$	(the output) $2\sin\left(x-\frac{\pi}{2}\right)+3$
$\frac{\pi}{2}$	0	0	0	3
π	$\frac{\pi}{2}$	1	2	5
$\frac{3\pi}{2}$	π	0	0	3
2π	$\frac{3\pi}{2}$	-1	-2	1
$\frac{5\pi}{2}$	2π	0	0	3

Observe that we now have five (x, y) coordinate pairs that describe a complete period of the graph of the function $y = 2\sin\left(x-\frac{\pi}{2}\right)+3$. These five coordinate pairs describe five points that are transformations of the original five famous points on the graph of $y = \sin(x)$. We can sketch a graph of one period of the function from this data.



[Example 2] (Students) Consider the function $y = 2\cos\left(\left(\frac{2}{3}\right)x - \frac{\pi}{3}\right)$. Think about what happens when you feed an input x into this function.

- Given input x,
- multiply the input by $\left(\frac{2}{3}\right)$ to get $\left(\frac{2}{3}\right)x$,
- then subtract $\frac{\pi}{3}$ from that to get $\left(\frac{2}{3}\right)x - \frac{\pi}{3}$,
- then take the cos of that result to get $\cos\left(\left(\frac{2}{3}\right)x - \frac{\pi}{3}\right)$,
- then finally multiply that result by 2 to get $2\cos\left(\left(\frac{2}{3}\right)x - \frac{\pi}{3}\right)$. This is the output.

These operations have been put into column headings for a table. Observe that the table has a total of six rows. One row is the column headings. The remaining five rows will be used for data pertaining to the five famous points on the cos graph.

(input) x	$\left(\frac{2}{3}\right)x$	$\left(\frac{2}{3}\right)x - \frac{\pi}{3}$	$\cos\left(\left(\frac{2}{3}\right)x - \frac{\pi}{3}\right)$	$2\cos\left(\left(\frac{2}{3}\right)x - \frac{\pi}{3}\right)$ (the output)

(a) Import the input/output data for the five famous points of the cos function into the appropriate pair of columns in the table.

(b) Fill out the rest of the columns as was done in Example 1.

(c) Make a graph of one period of $y = 2\cos\left(\left(\frac{2}{3}\right)x - \frac{\pi}{3}\right)$

[Example 3] (Students) The goal is to make a table of values for $y = -\sin(2\theta) + 3$ and then graph one period of the function.
(a) Make a table that has six rows, and as many columns as are needed to get from an input of θ to a final output of $y = -\sin(2\theta) + 3$. Fill in the first row (the column headings) as in Examples 1,2.
(b) Import the input/output data for the five famous points of the sin function into the appropriate pair of columns in your table.
(c) Fill out the rest of the columns as was done in Examples 1,2
(d) Graph one period of the function.

[Example 4] (Students) Make a table of values for $y = \frac{5}{2}\cos\left(\frac{3}{2}x\right)$ in the manner of Examples 1,2,3 and then graph one period of the function.