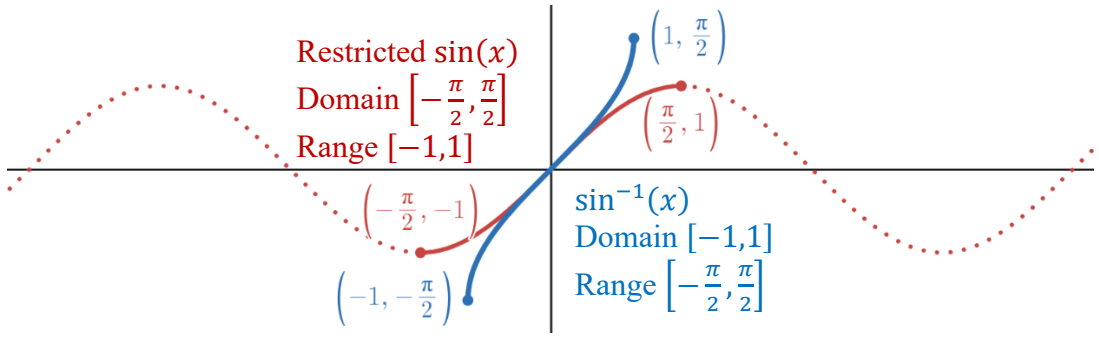
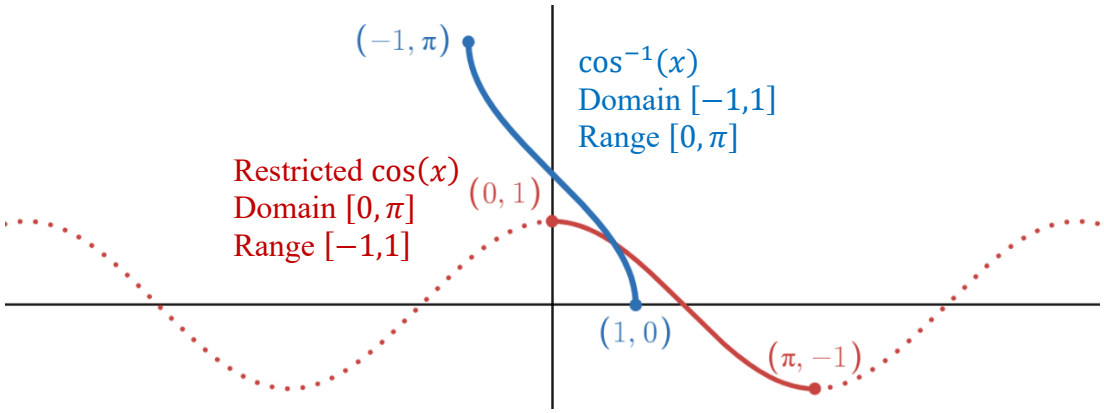
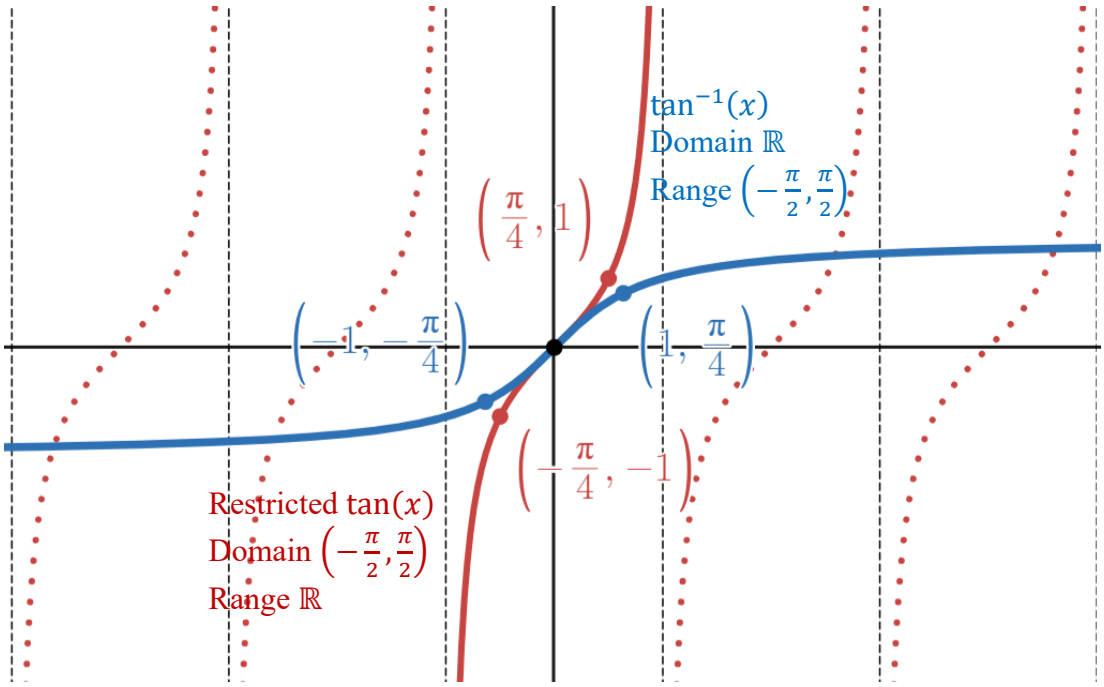
Handout for MATH 1300 (Barsamian) Lecture L35 Thu Nov 5, 2026

Section 4.7 The Inverse Trigonometric Functions

The regular trig functions $\sin(x)$, $\cos(x)$, $\tan(x)$ are *not one-to-one* because they fail the horizontal line test. Therefore, they do not have inverse functions. The ***restricted trig functions*** *are* one-to-one and so they *do* have inverse functions, which are called the ***inverse trig functions*** and are denoted $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$.







It is important to remember that the ***inverse trig functions*** are the inverse functions for the ***restricted trig functions***, *not* for the regular trig functions. This means that one must be careful when using the ***cancellation laws***, paying attention to the domains.

Cancellation Laws

for sin

$\sin^{-1}(\sin(x)) = x$ for all x in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin(\sin^{-1}(x)) = x$ for all x in $[-1,1]$

for cos

$\cos^{-1}(\cos(x)) = x$ for all x in $[0, \pi]$

$\cos(\cos^{-1}(x)) = x$ for all x in $[-1,1]$

for tan

$\tan^{-1}(\tan(x)) = x$ for all x in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan(\tan^{-1}(x)) = x$ for all x in $\mathbb{R}$

[1] Examples involving the Cancellation Laws (A unit circle drawing may be helpful.)

(a) $\sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right) =$

(b) $\cos^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right) =$

(c) $\tan^{-1}\left(\tan\left(\frac{\pi}{3}\right)\right) =$

(d) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) =$

(e) $\cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right) =$

(f) $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right) =$

[2] Examples involving mixed functions (A right triangle drawing may be helpful.)

(a) $\tan(\cos^{-1}(x)) =$

(b) $\cos(\tan^{-1}(x)) =$

[3] (for students to do at board)

(1) $\sin^{-1}\left(\frac{1}{2}\right)$

(2) $\sin^{-1}\left(\frac{3}{2}\right)$

(3) $\tan^{-1}\left(-\sqrt{3}\right)$

(4) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(5) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(6) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(7) $\sin\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

(8) $\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$

(9) $\sin^{-1}\left(\sin\left(\frac{5\pi}{3}\right)\right)$

(10) $\sin\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$

(11) $\cos\left(\sin^{-1}\left(-\frac{2}{7}\right)\right)$

(12) $\tan\left(\sin^{-1}\left(-\frac{\sqrt{5}}{3}\right)\right)$

(13) $\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right)$

(14) $\tan^{-1}\left(\tan\left(\frac{5\pi}{3}\right)\right)$