

## **Handout: Using The Squeeze Theorem (Stewart Section 1.4)**

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### **The Squeeze Theorem as Presented in the Book**

If  $f(x) \leq g(x) \leq h(x)$  when  $x$  is near  $a$  (except possibly at  $a$ ) and

$$\lim_{x \rightarrow a} f(x) = L = \lim_{x \rightarrow a} h(x)$$

then

$$\lim_{x \rightarrow a} g(x) = L$$

### **The Squeeze Theorem presented another way**

If we know that all this stuff is true (these are the “hypotheses”):

- There is some real number  $a$ .
- There is a function  $f(x)$ .
- There is a function  $g(x)$ .
- There is a function  $h(x)$ .
- The functions  $f, g, h$  satisfy  $f(x) \leq g(x) \leq h(x)$  when  $x$  is near  $a$  (except possibly at  $a$ )
- The function  $f$  has limit  $\lim_{x \rightarrow a} f(x) = L$ .
- The function  $h$  has the same limit  $\lim_{x \rightarrow a} h(x) = L$ .

Then we are allowed to say these words (this is the “conclusion”):

- “The function  $g$  has limit  $\lim_{x \rightarrow a} g(x) = L$ .”

When we “use” this theorem (or any other), we start by verifying explicitly that all of the hypotheses are satisfied in our specific situation. If they are satisfied, then we write the conclusion, adapted to our specific situation.

A worksheet for this process is provided on the back of this page.

### Squeeze Theorem Worksheets

| Generic Hypotheses                                                                                                            | Our Specific Hypotheses                                                                                                                      |
|-------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| the real number $a$                                                                                                           |                                                                                                                                              |
| the function $f(x)$                                                                                                           |                                                                                                                                              |
| the function $g(x)$                                                                                                           |                                                                                                                                              |
| the function $h(x)$                                                                                                           |                                                                                                                                              |
| verification that functions $f, g, h$ satisfy<br>$f(x) \leq g(x) \leq h(x)$<br>when $x$ is near $a$ (except possibly at $a$ ) |                                                                                                                                              |
| Function $f$ has limit $\lim_{x \rightarrow a} f(x) = L$ .                                                                    |                                                                                                                                              |
| Function $h$ has limit $\lim_{x \rightarrow a} h(x) = L$ .                                                                    |                                                                                                                                              |
| Generic Conclusion                                                                                                            | Our Specific Conclusion                                                                                                                      |
| Function $g(x)$ has limit $\lim_{x \rightarrow a} g(x) = L$ .                                                                 | <p>Function ____(<math>x</math>) = _____</p> <p>has limit <math>\lim_{x \rightarrow \_\_\_\_\_\_} \_\_\_\_\_\_(x) = \_\_\_\_\_\_.</math></p> |

| Generic Hypotheses                                                                                                            | Our Specific Hypotheses                                                                                                                      |
|-------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| the real number $a$                                                                                                           |                                                                                                                                              |
| the function $f(x)$                                                                                                           |                                                                                                                                              |
| the function $g(x)$                                                                                                           |                                                                                                                                              |
| the function $h(x)$                                                                                                           |                                                                                                                                              |
| verification that functions $f, g, h$ satisfy<br>$f(x) \leq g(x) \leq h(x)$<br>when $x$ is near $a$ (except possibly at $a$ ) |                                                                                                                                              |
| Function $f$ has limit $\lim_{x \rightarrow a} f(x) = L$ .                                                                    |                                                                                                                                              |
| Function $h$ has limit $\lim_{x \rightarrow a} h(x) = L$ .                                                                    |                                                                                                                                              |
| Generic Conclusion                                                                                                            | Our Specific Conclusion                                                                                                                      |
| Function $g(x)$ has limit $\lim_{x \rightarrow a} g(x) = L$ .                                                                 | <p>Function ____(<math>x</math>) = _____</p> <p>has limit <math>\lim_{x \rightarrow \_\_\_\_\_\_} \_\_\_\_\_\_(x) = \_\_\_\_\_\_.</math></p> |