

Handout: Linearizations, Linear Approximations, and Differentials (Stewart Section 2.8)

Definition of the Linearization

Words: The *linearization* of $f(x)$ at $x = a$

Meaning: The function $L(x)$ defined by the equation

$$L(x) = f(a) + f'(a)(x - a)$$

Graphical Significance: $L(x)$ describes the line that is tangent to the graph of $f(x)$ at $x = a$.

Method for Finding a Linear Approximation

Given: a function $f(x)$ and a hard x value called $\hat{x}$. (That is, it is not easy, or maybe even not possible, to compute $f(\hat{x})$ exactly by hand.)

Goal: Find an *approximation* for $f(\hat{x})$.

Steps:

- 1) Identify the function $f(x)$
- 2) Identify the hard x value, called $\hat{x}$.
- 3) Identify an easy nearby $x = a$ such that $f(a)$ is easy to compute.
- 4) Build the *linearization* of $f(x)$ at a . That is, build the function

$$L(x) = f(a) + f'(a)(x - a)$$

- 5) Use the *linearization* to compute the number $L(\hat{x})$. That is, compute

$$L(\hat{x}) = f(a) + f'(a)(\hat{x} - a)$$

(This should be an easy calculation.) This number $L(\hat{x})$ is the desired *approximation* for $f(\hat{x})$. It is called the *linear approximation* for $f(\hat{x})$.

Differentials

Consider function f and two known x values x_1 and x_2 . How does the value of $y = f(x)$ change when x changes from x_1 to x_2 ?

- change in x : $\Delta x = x_2 - x_1 = \text{change in } x$
- exact change in y : $\Delta y = f(x_2) - f(x_1)$ (conceptually simple but may be hard to compute!)
- approximate change in y : $dy = f'(x_1) \cdot \Delta x$ (the differential of f at x_1)(easier to compute)