Handout: Graphing Strategy (concepts from Stewart Section 4.4)

General Idea: Start with simplest, least sophisticated analysis, and proceed to more sophisticated analysis. If a task is not easy, then maybe skip that task.

Step 1. Analyze $f(x)$.

- What is the domain of $f(x)$? Where is $f(x)$ continuous?
- Find the (x, y) coordinates of the y-intercept.
- Factor $f(x)$, and use the factorization to find the *partition numbers of* f. (These are the x values where $f(x) = 0$ or $f(x)$ is discontinuous.)
- Find the (x, y) coordinates of all x intercepts.
- Make a sign chart for f to show where f is positive, negative, zero, undefined.
- Write the line equations of any vertical asymptotes.
- Does the graph of $f(x)$ have any symmetry or any periodicity?
- Determine the end-behavior (Are there horizontal asymptotes? Or do the ends of the graph go up or down?) by finding $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. (May require L'Hospital's Rule!)

Step 2. Analyze $f'(x)$.

- Find $f'(x)$, factor it, and use the factorization to find the *partition numbers of* f'. (These are the x values where $f'(x) = 0$ or $f'(x)$ is discontinuous.)
- Construct a sign chart for $f'(x)$ and use it to determine the x coordinates where graph of f has a horizontal tangent line and the intervals on which f is increasing and decreasing.
- Find the (x, y) coordinates of all local maxima and minima of $f(x)$.

Step 3. Analyze $f''(x)$.

- Find $f''(x)$, factor it, and use the factorization to find the *partition numbers of* f''. (These are the x values where $f''(x) = 0$ or f'' is discontinuous.)
- Make a sign chart for $f''(x)$ to determine where f is concave up and concave down.
- Find the (x, y) coordinates of all inflection points of $f(x)$.

Step 4: Sketch the graph of f.

- Draw any asymptotes as dotted lines, and label them with their line equations.
- Plot the axis intercepts, local maxima and minima, and inflection points, and label them with their (x, y) coordinates.
- Using the other information from steps 1, 2, and 3, draw the graph.

<u>**Handout: Three Rational Functions and their Graphs**</u>

<table>
<tr><th>Function and its derivatives</th><th>graph</th></tr>
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$f(x) = \frac{100}{x^2+25}$

$f'(x) = \cdots = -\frac{200x}{(x^2+25)^2}$

$f''(x) = \cdots = \frac{200(3x^2-25)}{(x^2+25)^3}$

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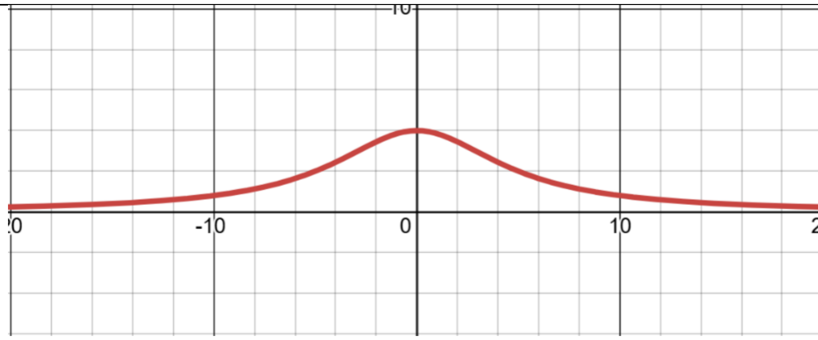

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$g(x) = \frac{100}{x^2-25}$

$g'(x) = -\frac{200x}{(x^2-25)^2}$

$g''(x) = \frac{200(3x^2+25)}{(x^2-25)^3}$

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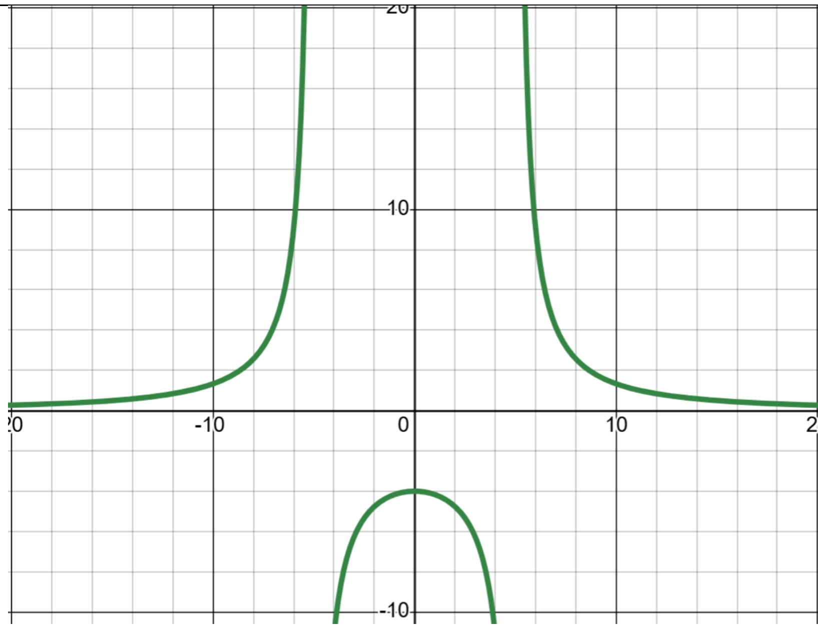

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$h(x) = \frac{100x}{x^2+25}$

$h'(x) = -\frac{100(x^2-25)}{(x^2+25)^2}$

$h''(x) = \frac{200x(x^2-75)}{(x^2+25)^3}$

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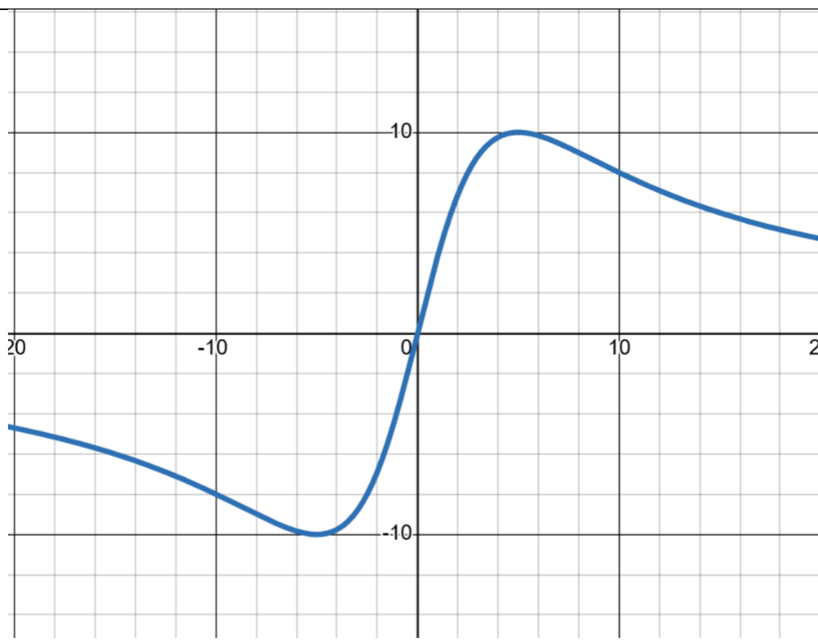

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