

Handout: The Substitution Rule (Stewart Section 5.5)

Remember that the *Chain Rule for Derivatives* is used for taking the *derivative* of *nested functions*:

$$\text{Chain Rule for Derivatives: } \frac{d}{dx} \text{outer}(\text{inner}(x)) = \text{outer}'(\text{inner}(x)) \cdot \text{inner}'(x)$$

The goal now is to find the *general antiderivative* of a function $f(x)$ that involves a *nested function*.

That is, we wish to find the *indefinite integral* $\int f(x) dx$ where the integrand $f(x)$ involves a nested function. This is not always possible. But sometimes it is, using the *Substitution Method*.

The Substitution Rule for finding the *indefinite integral* $F(x) = \int f(x) dx$

where the integrand $f(x)$ involves a *nested function*.

Step 1 Identify the inner function and call it u . Write the equation $\text{inner}(x) = u$ to introduce the single letter u to represent the inner function. Circle the equation.

Step 2 Build the equation $dx = \frac{1}{u'} du$. To do this, first find u' , then use it to build equation $dx = \frac{1}{u'} du$. Circle the equation.

Step 3 Substitute, Cancel, Simplify. In steps (1) and (2) you have two circled equations. **Substitute** these into the integrand of your indefinite integral. **Cancel** as much as possible and **simplify** by using the *Constant Multiple Rule*. The result should be a new basic integral involving just the variable u . (See **Remarks about Step 3** below.)

Step 4 Integrate. Find the new indefinite integral by using the indefinite integral rules. The result should be a *function form* involving just the variable u and $+C$.

Step 5 Substitute Back. Substitute $u = \text{inner}(x)$ into your function from Step (4) The result will be a new function form involving just the variable x and the $+C$. This is the $F(x)$ that we seek. Present the result clearly as $F(x) = \text{BLAH}$ and circle it.

Remarks about Step 3: The result of **Step 3** should be a new indefinite integral with an integrand that is a function involving the variable u . There are three important things to check at the end of **Step 3**:

- There should be no x in the new indefinite integral. It should involve only u .
- The new indefinite integral should *not* involve a *nested function*, and it should be a *basic integral* that can be integrated using our indefinite integral rules.
- If the above two items are not satisfied, then either you made a mistake, or the original integral might be one for which the Substitution Method cannot be used.

Examples Using the Substitution Rule

Apply the Substitution Rule to these integrals. (Hint: One doesn't work!)

$$[1] \int e^{(kx)} dx$$

$$[2] \int x e^{(-x^2)} dx$$

$$[3] \int e^{(-x^2)} dx$$

$$[4] \int \frac{1}{7 - 13x} dx$$

$$[5] \int \frac{(\ln x)^5}{x} dx$$