Discussion Activity about Substitution, to Accompany H74 (Section 5.2 Concepts)

Frick and Frack are arguing about $\int e^{(x^2)}dx$

Frick says that $\int e^{(x^2)}dx = e^{(x^2)} + C$ because $\int e^{(x)}dx = e^{(x)} + C$

Frack says that $\int e^{(x^2)}dx = e^{\left(\frac{x^3}{3}\right)} + C$ because you must also integrate the x^2.

Neither can provide any other explanation for their own answer.

Neither will accept the other's answer, but neither can explain why the other's answer is wrong.

(a) Who is right or wrong? Explain clearly.

(b) Show clearly what happens when the ***substitution method*** is used on the integral $\int e^{(x^2)}dx$

For Reference, the ***substitution method*** is shown on the next two pages.

Two blank pages are provided after that, for writing notes.

The Substitution Method for finding the *indefinite integral*

$$F(x) = \int f(x)\,dx$$

where the integrand $f(x)$ involves a *nested function*.

Step 1 Identify the inner function and call it $\boldsymbol{u}$. Write the equation $inner(x) = u$ to introduce the single letter u to represent the inner function. Circle the equation.

Step 2 Build the equation $\boldsymbol{dx = \frac{1}{u'}du}$. To do this, first find u', then use it to build equation $dx = \frac{1}{u'}du$. Circle the equation.

Step 3 Substitute, Cancel, Simplify. In steps (1) and (2) you have two circled equations. Substitute these into the integrand of your indefinite integral. Cancel as much as possible and simplify by using the *Constant Multiple Rule*. The result should be a new basic integral involving just the variable u. (See **Remarks about Step 3** on the next page.)

Step 4 Integrate. Find the new indefinite integral by using the indefinite integral rules. The result should be a function involving just the variable u (with constant of integration $+C$).

Step 5 Substitute Back. Substitute $u = inner(x)$ into your function from Step (4) The result will be a new function of just the variable x. (Be sure to include the constant of integration $+C$ in your result.) This is the $F(x)$ that we seek.

Remarks about Step 3

Note that the result of **Step 3** should be a new indefinite integral with an integrand that is a function involving the variable u. There are three important things to check at the end of **Step 3**:

- There should be no x in the new indefinite integral. It should involve only u.
- The new indefinite integral should *not* involve a *nested function*, and it should be a *basic integral* that can be integrated using our indefinite integral rules.
- If the above two items are not satisfied, then either you made a mistake, or the original integral might be one for which the Substitution Method cannot be used.