Section 2.2 Limits Involving Infinity (Barnett Book Section 2.2)

In this video, we will consider a graphical approach to limits involving infinity.

Definition of Infinite Limits

Symbol: $\lim_{x \to c} f(x) = \infty$

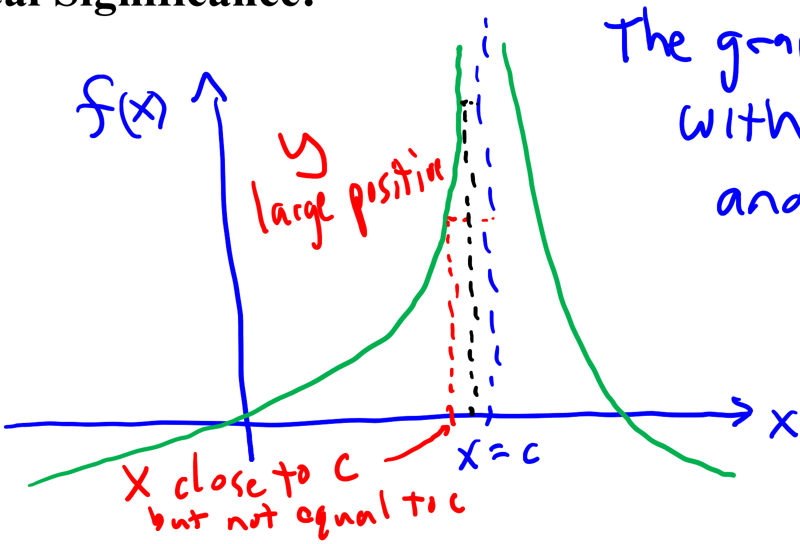
Spoken: The limit, as x approaches c, of $f(x)$ is infinity.

Usage: x is a variable, f is a function, c is a real number constant.

Meaning: As x gets closer & closer to c, but not equal to c, the values of $f(x)$ get more & more positive without bound.

Graphical Significance: The graph of $f(x)$ has a vertical asymptote with line equation $x = c$ and the graph is going up along the asymptote on both left and right sides

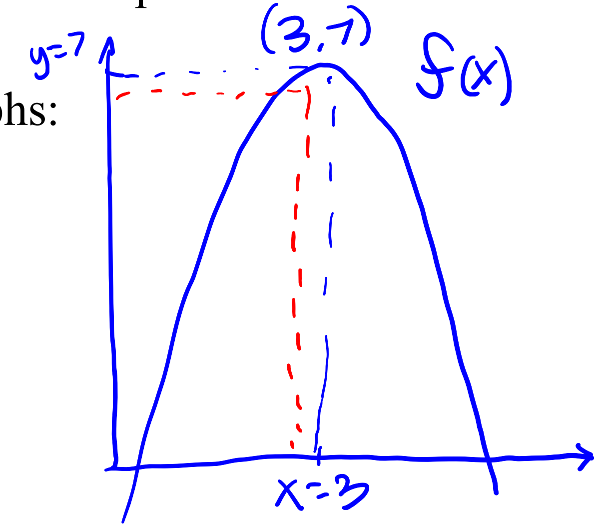
Picture:

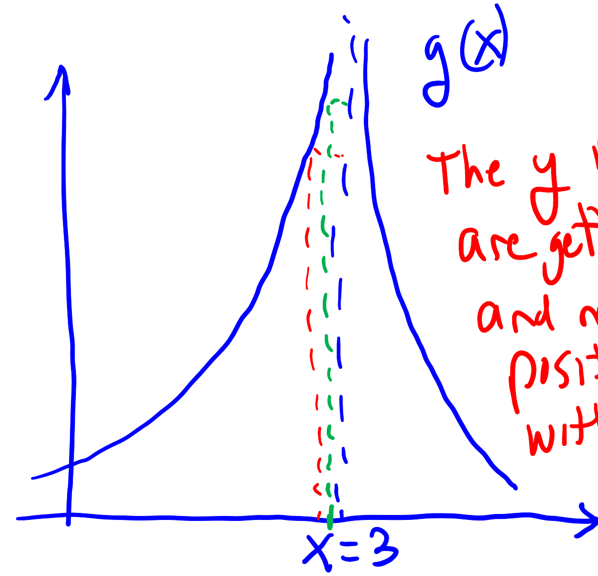


What is the significance of the phrase *without bound*?

Consider these two graphs:

$\lim_{x\to 3} f(x) = 7$





The y values are getting more and more positive without bound

In both graphs, as x approaches 3, the y values get *more and more positive*. But in the graph on the left, the y values are getting more and more positive and *approaching 7*, while in the graph on the right, the y values are getting more and more positive *without bound*.

Remark: (a very important point) We have revised the definition of limit! In Section 2.1, the book says that for the drawing shown above, the limit does not exist, but we write the symbol $\lim_{x\to 3} \cancel{f}\, g(x) = \infty$. That's confusing and misses the point. The notation and terminology of limits is simply a helpful shorthand for trends that are often observed in functions and their graphs. What we have done in Section 2.2 is that we have expanded the definition of limit to include another type of trend. With the old definition of limit (from Section 2.1), the limit does not exist in the drawing above. With the new, expanded definition of limit (from Section 2.2), the limit does exist and it is infinity. In section 2.1, we would say $\lim_{x\to 3} g(x)$ Does not exisT

In section 2.2, we say that $\lim_{x\to 3} g(x) = \infty$.

Obvious variations:

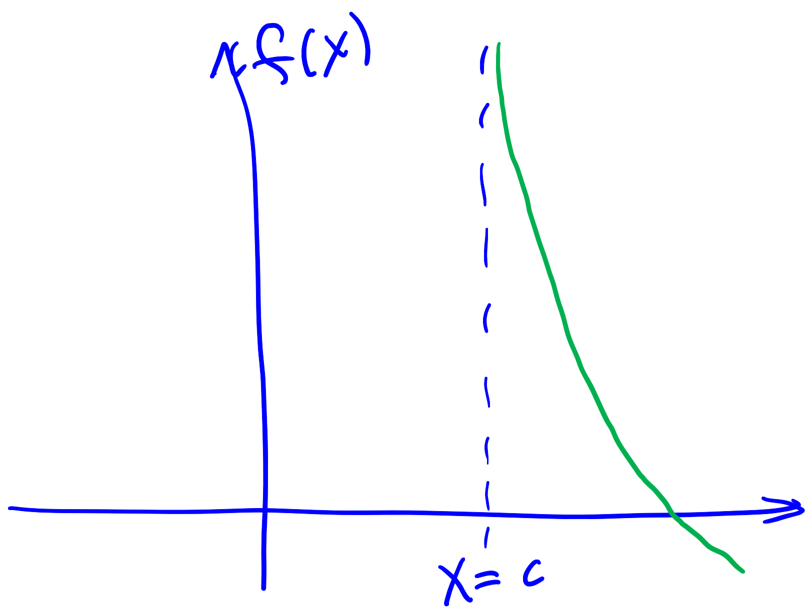
One-Sided Infinite Limits

Symbol: $\lim_{x \to c^+} f(x) = \infty$

Spoken: The limit, as x approaches c from the right, of f(x) is infinity.

Meaning: As x gets closer and closer to c but greater than c, the values of f(x) get more and more positive without bound.

Picture:



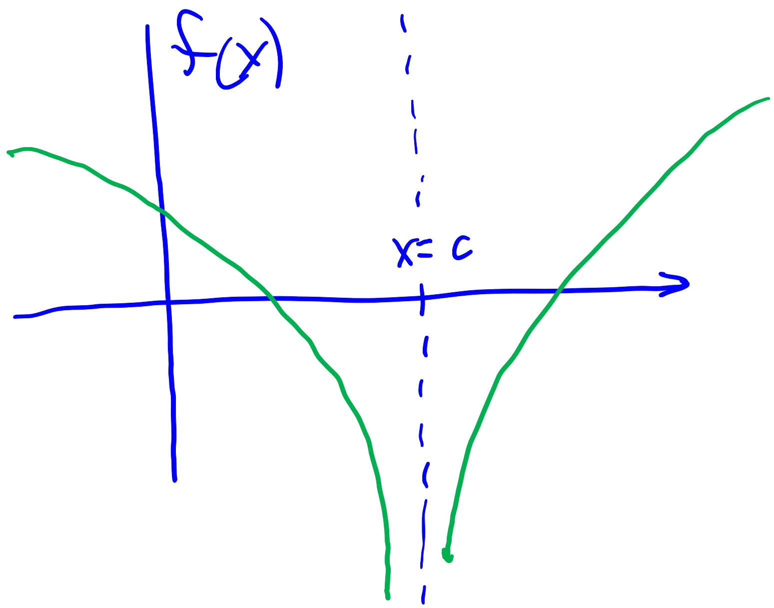
Negative Infinite Limits

Symbol: $\lim_{x \to c} f(x) = -\infty$

Spoken: The limit, as x approaches c, of $f(x)$ is negative infinity.

Meaning: As x approaches c, the values of $f(x)$ get more and more negative, without bound,

Picture:



Observation: With the definition of infinite limits, we now have a shorthand notation to describe the situation where the y values of a function are getting more and more positive (or negative) without bound.

Definition of Limits at Infinity

symbol: $\lim_{x \to \infty} f(x) = b$

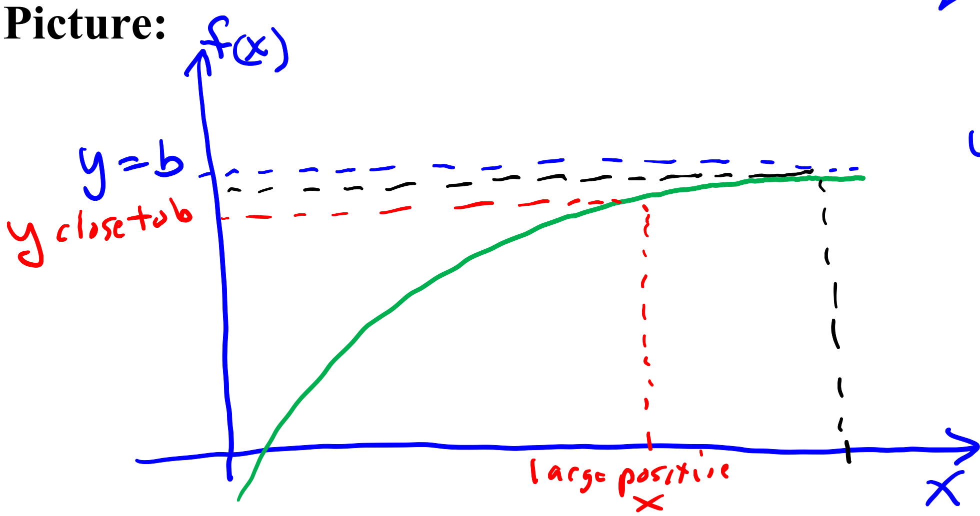
spoken: The limit, as x goes to infinity, of f(x) is b.

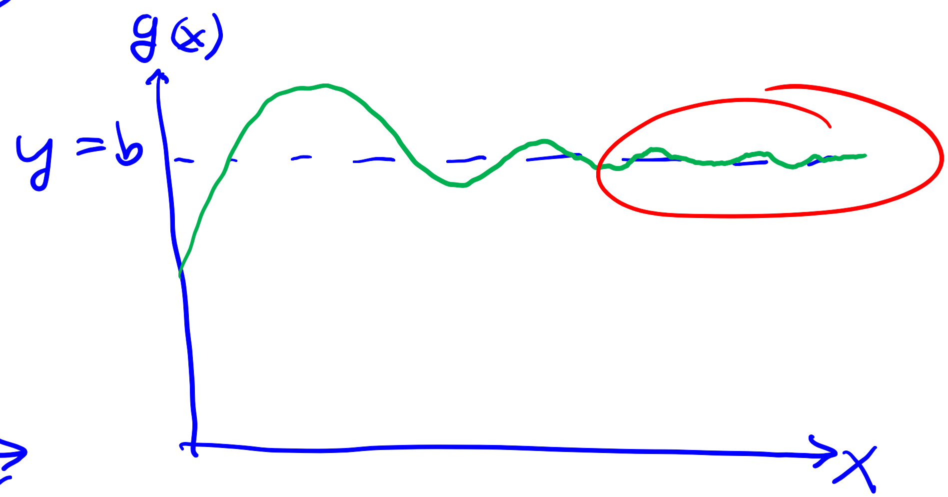
Usage: x is a variable, f is a function, b is a real number constant.

Meaning: As x gets more and more positive, without bound, the values of f(x) get closer and closer to b (and may equal b)

Graphical Significance: The graph has a horizontal asymptote on the right with line equation y=b.

Picture:





Obvious variations:

Definition of Limit at Negative Infinity

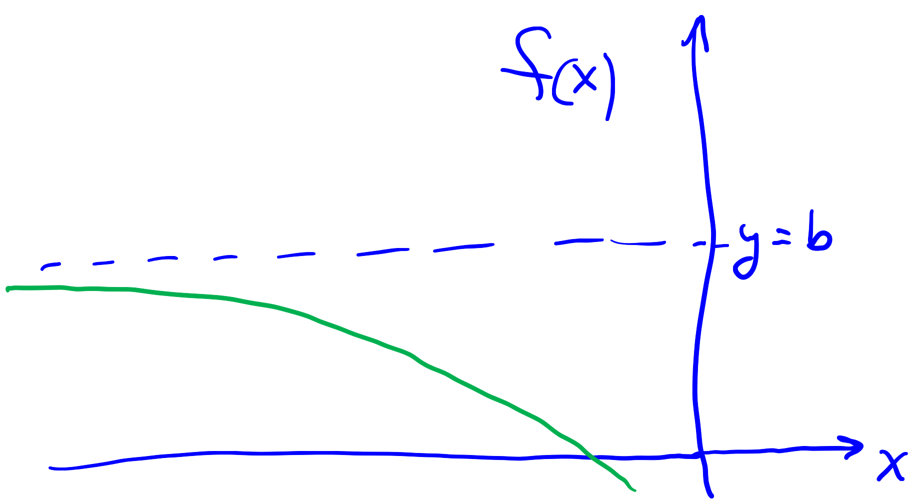
symbol: $\lim_{x \to -\infty} f(x) = b$

spoken: The limit, as x goes to negative infinity, of f(x) is b.

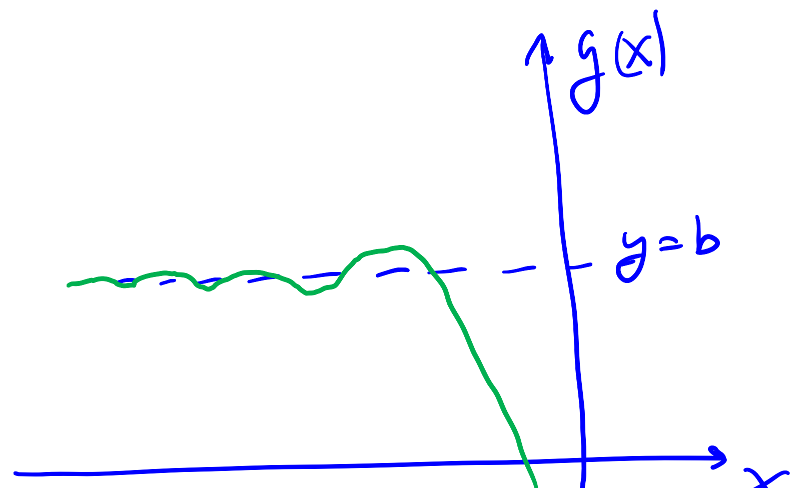
Meaning: As x gets more + more negative, without bound, the values of f(x) get closer + closer to b (and may equal b.)

Graphical Significance: The graph has a horizontal asymptote on the left with line equation y=b.

Picture:



or



We can combine the idea of infinite limits and limits at infinity in the obvious way

Definition of Infinite Limits at Infinity

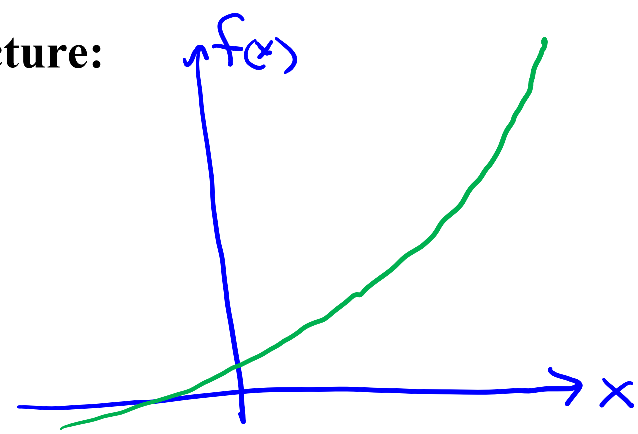
symbol: $\lim_{x \to \infty} f(x) = \infty$

spoken: The limit, as x goes to infinity, of $f(x)$ is infinity.

Meaning: As x gets more & more positive, without bound, the values of $f(x)$ get more & more positive, without bound.

Graphical Significance: The right end of the graph goes up.

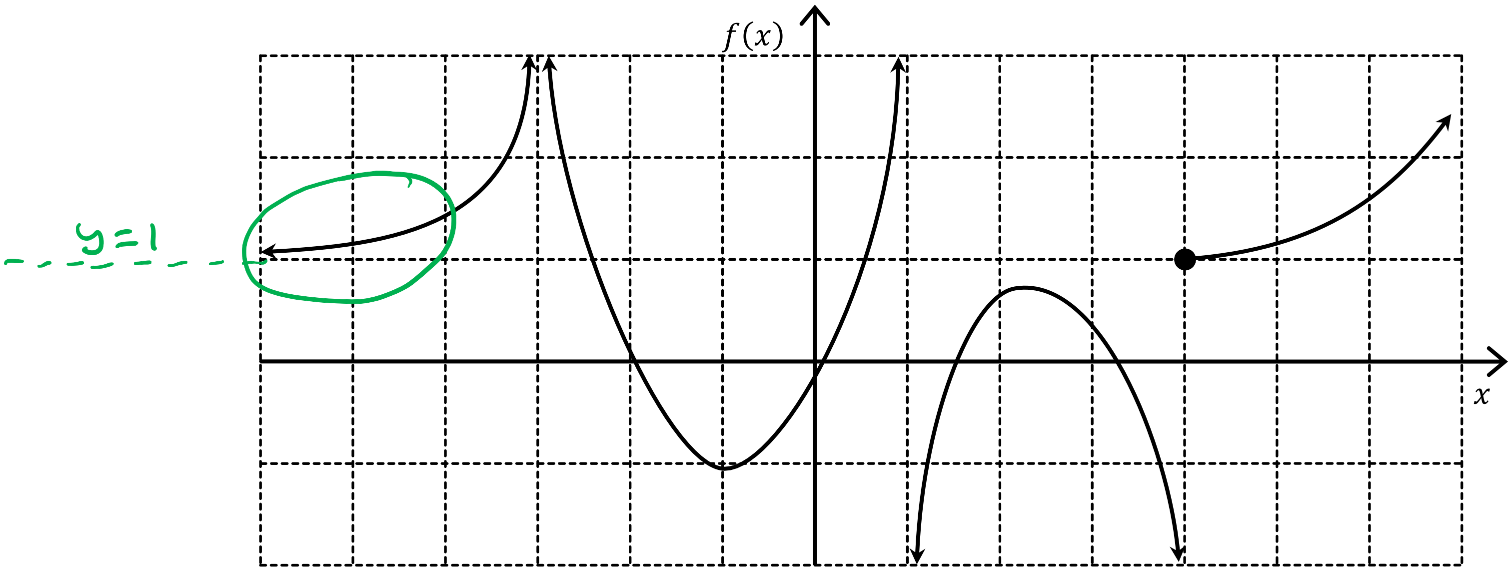
Picture:



Remark: The term *limit at infinity* (or *limit at negative infinity*) refers to what the right (or left) end of the graph is doing. This is called the *end behavior* of the graph.

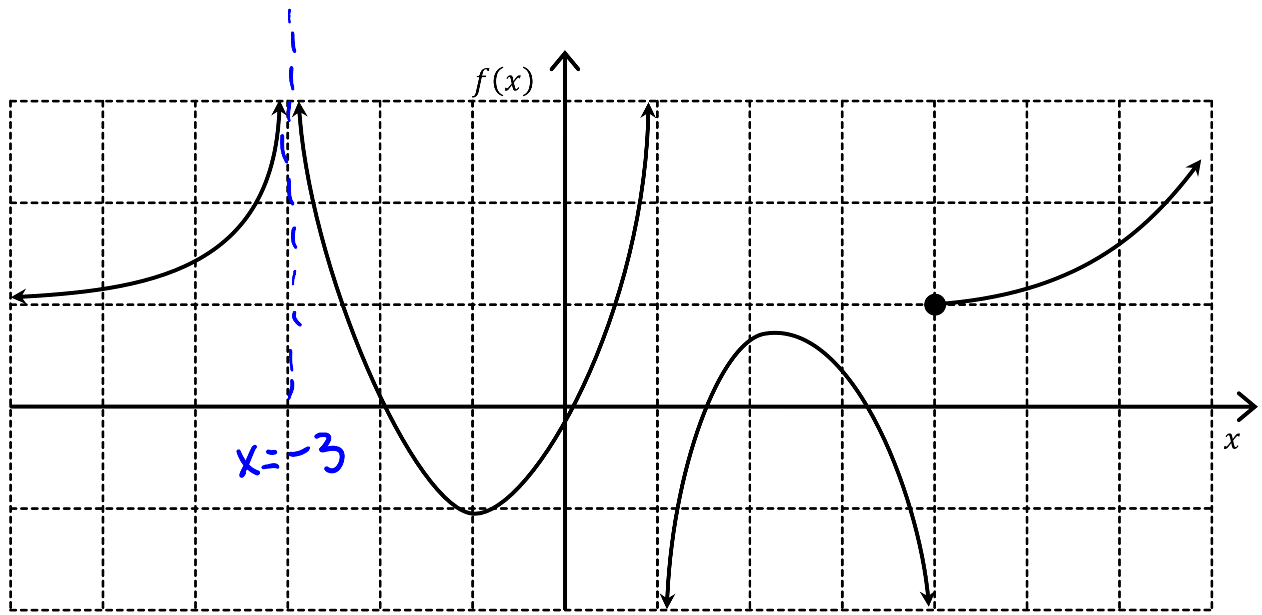
Limits Involving Infinity for a Function Given by a Graph (Section 2.2)

Use the graph to answer the questions that follow.



(A) $\lim_{x \to -\infty} f(x) =$ 1 because the graph has a horizontal asymptote on the left with line equation $y = 1$.

left end behavior

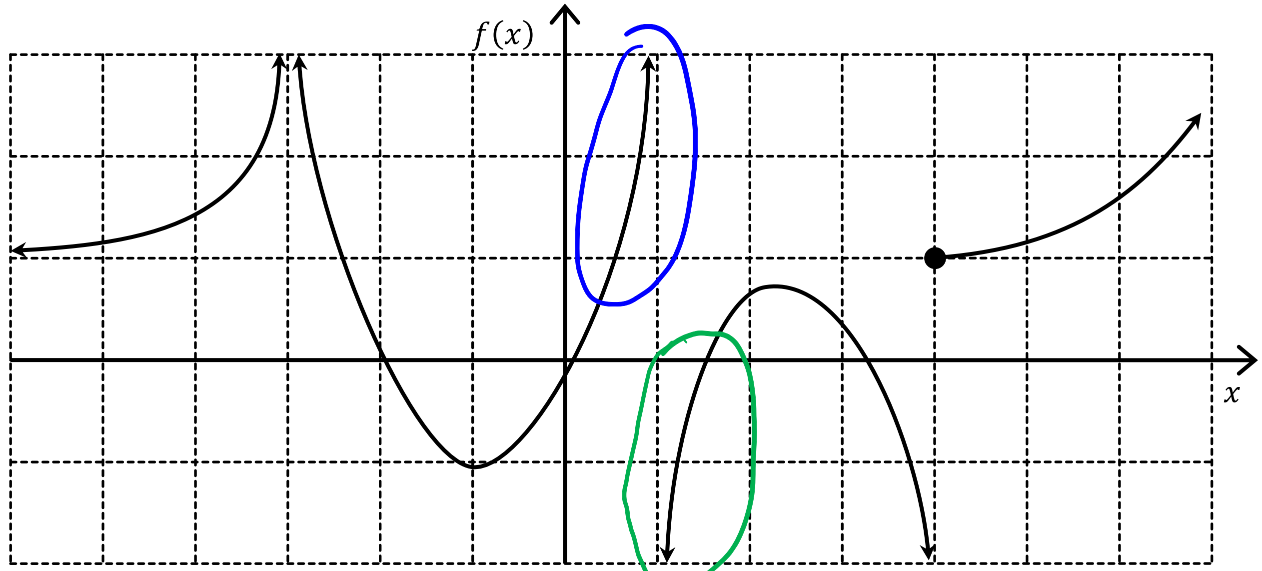


(B) $\lim_{x \to -3} f(x) = \infty$ because as x approaches -3, the values of f(x) get more + more positive, without bound.

Equivalently: The graph of f(x) has a vertical asymptote with line equation x=-3, and the graph of f(x) goes up along both sides of the asymptote.

(C) $f(-3) =$ DNE

There is no point on the graph with x=3



(D) $\lim_{x \to 1^-} f(x) = \infty$

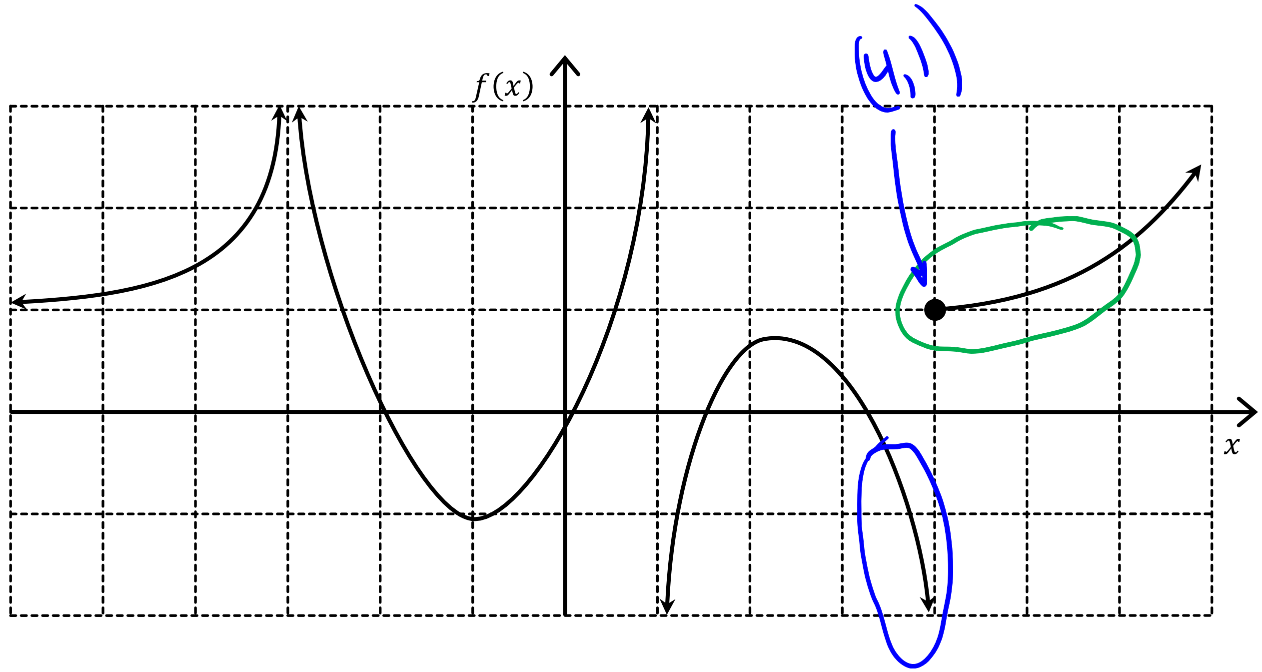
The graph has a vertical asymptote with equation $x=1$, and the graph goes up along the left side of the asymptote.

(E) $\lim_{x \to 1^+} f(x) = -\infty$

The graph goes down along the right side of the asymptote

(F) $\lim_{x \to 1} f(x) =$ Does not exist, because the left & right limits don't match.

(G) $f(1) =$ Does not exist, because there is no point on the graph with $x=1$.

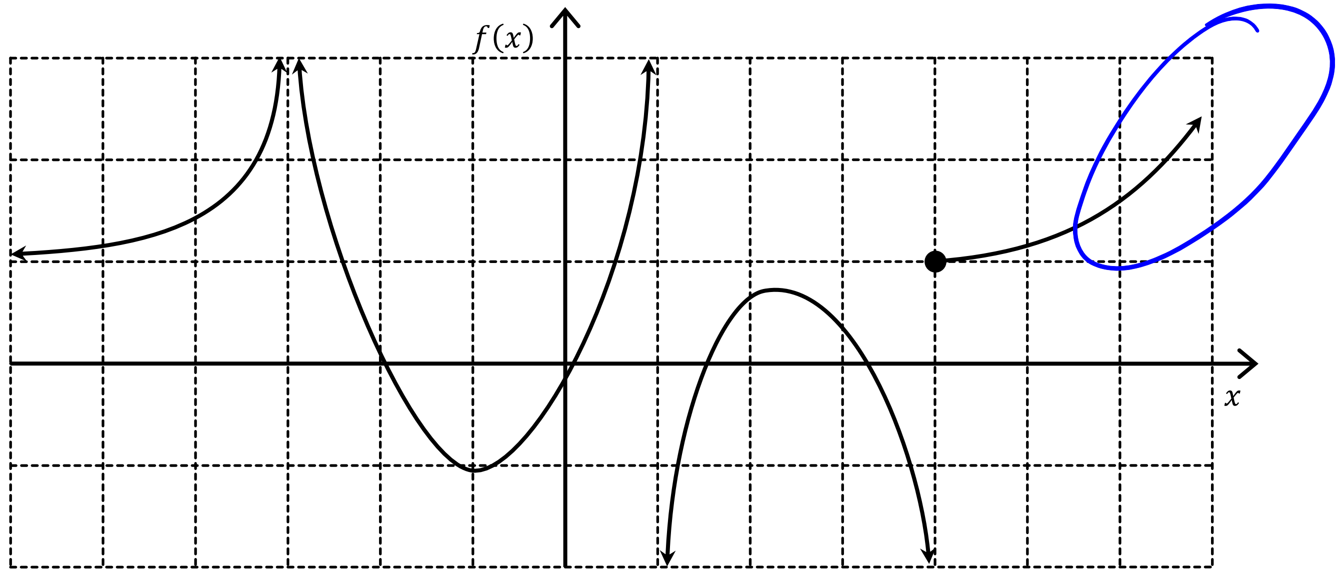


(H) $\lim_{x\to 4^-} f(x) = -\infty$ The graph has a vertical asymptote with line equation $x = 4$ and the graph goes down along the left side of the asymptote

(I) $\lim_{x\to 4^+} f(x) = 1$ The graph is heading for the location $(x, y) = (4, 1)$ from the right.

(J) $\lim_{x\to 4} f(x) =$ DNE because the left and right limits don't match.

(K) $f(4) = 1$ because there is a point on the graph at $(x, y) = (4, 1)$



$$(L)\ \lim_{x\to\infty} f(x) = \infty$$

The right end of the graph goes up without bound.

Summarizing Limit Terminology from Sections 2.1 and 2.2

As mentioned throughout this video and in previous videos, limit terminology is simply an *abbreviation* for certain kinds of *trends* that sometimes occur in the x values and y values for particular functions. The corresponding limit notation is an even more an even more condensed abbreviation. Here's a summary of limit *notation* and the corresponding *trends*.

Limit Notation Introduced in Section 2.1

		Trend in the *y*-values	
		y values approach the number L (and may equal L)	y values do anything else
Trend in the *x*-values	x approaches the number c from the left	$\lim_{x\to c^-} f(x) = L$ (section 2.1)	$\lim_{x\to c^-} f(x)\ DNE$
	x approaches the number c from the right	$\lim_{x\to c^+} f(x) = L$ (section 2.1)	$\lim_{x\to c^+} f(x)\ DNE$
	x approaches the number c	$\lim_{x\to c} f(x) = L$ (section 2.1)	$\lim_{x\to c} f(x)\ DNE$
	x values do anything else	no terminology in Section 2.1	no terminology in Section 2.1

Observation: In Section 2.1, a limit can only be a real number, occurring at a particular x value. The limit symbol can only mean that the graph is heading for a location $(x, y) = (c, L)$

Limit Notation Introduced in Section 2.2

		Trend in the *y*-values			
		y values approach the number *L* (and may equal *L*)	*y* values grow negative, without bound	*y* values grow positive, without bound	*y* values do anything else
Trend in the *x*-values	*x* approaches the number *c* from the left	$\lim_{x\to c^-} f(x) = L$ (section 2.1)	$\lim_{x\to c^-} f(x) = -\infty$ (section 2.2)	$\lim_{x\to c^-} f(x) = \infty$ (section 2.2)	$\lim_{x\to c^-} f(x)\ DNE$
	x approaches the number *c* from right	$\lim_{x\to c^+} f(x) = L$ (section 2.1)	$\lim_{x\to c^+} f(x) = -\infty$ (section 2.2)	$\lim_{x\to c^+} f(x) = \infty$ (section 2.2)	$\lim_{x\to c^+} f(x)\ DNE$
	x approaches the number *c*	$\lim_{x\to c} f(x) = L$ (section 2.1)	$\lim_{x\to c} f(x) = -\infty$ (section 2.2)	$\lim_{x\to c} f(x) = \infty$ (section 2.2)	$\lim_{x\to c} f(x)\ DNE$
	x grows negative, without bound	$\lim_{x\to -\infty} f(x) = L$ (section 2.2)	$\lim_{x\to -\infty} f(x) = -\infty$ (section 2.2)	$\lim_{x\to -\infty} f(x) = \infty$ (section 2.2)	$\lim_{x\to -\infty} f(x)\ DNE$
	x grows positive, without bound	$\lim_{x\to \infty} f(x) = L$ (section 2.2)	$\lim_{x\to \infty} f(x) = -\infty$ (section 2.2)	$\lim_{x\to \infty} f(x) = \infty$ (section 2.2)	$\lim_{x\to \infty} f(x)\ DNE$
	x values do anything else	no terminology in Section 2.1 or 2.2	no terminology in Section 2.1 or 2.2	no terminology in Section 2.1 or 2.2	no terminology in Section 2.1 or 2.2

Observation: In Section 2.2, more limit terminology and limit symbols are introduced. These are are just shorthand notation for additional kinds of trends that are often seen in functions and their graphs.

End of Video