Topic for this Video: Introduction to Continuity

Reading:

- **General:** Section 2.3 Continuity
- **More specifically:** Pages 118 – 121, including Example 1

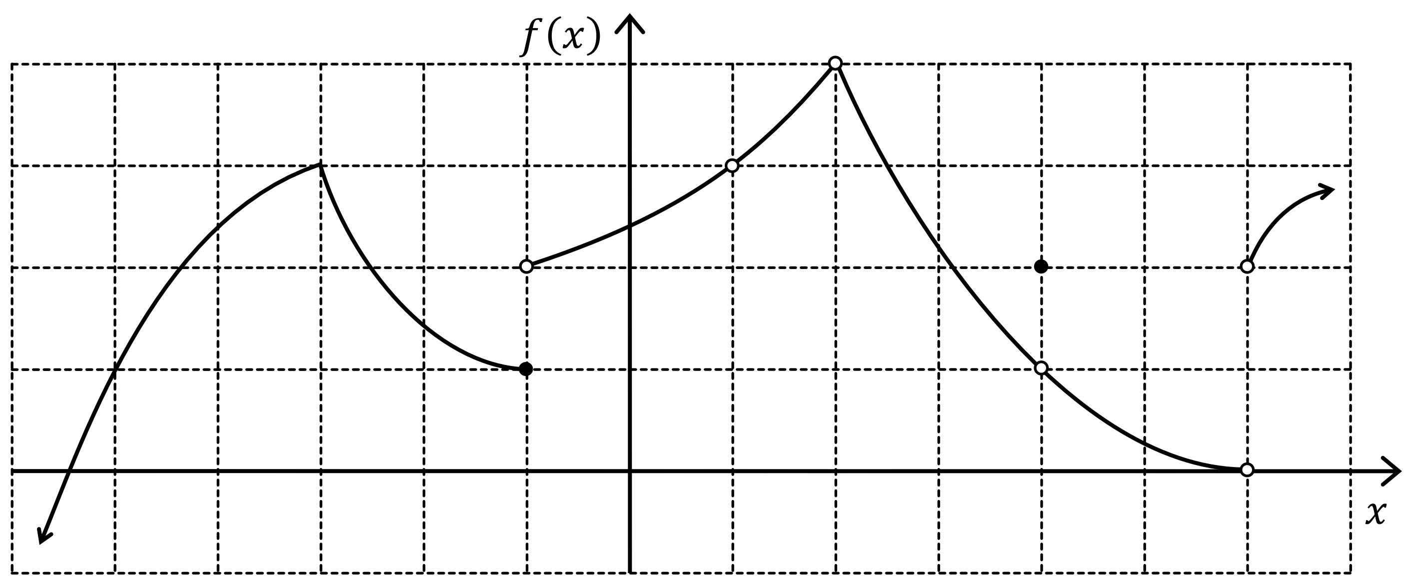
Homework:

H16 Given graph of f, find limit and continuity behavior (2.3#19,20,27)

Recall the Examples that we did in the Video for Homework H02 involving finding limits for a function given by a graph.

Here is the graph:

<u>Graph from Video for H02: Limits for a Function Given by a Graph (Section 2.1)</u>



Observe that the graph has holes, jumps, and points in the wrong places.

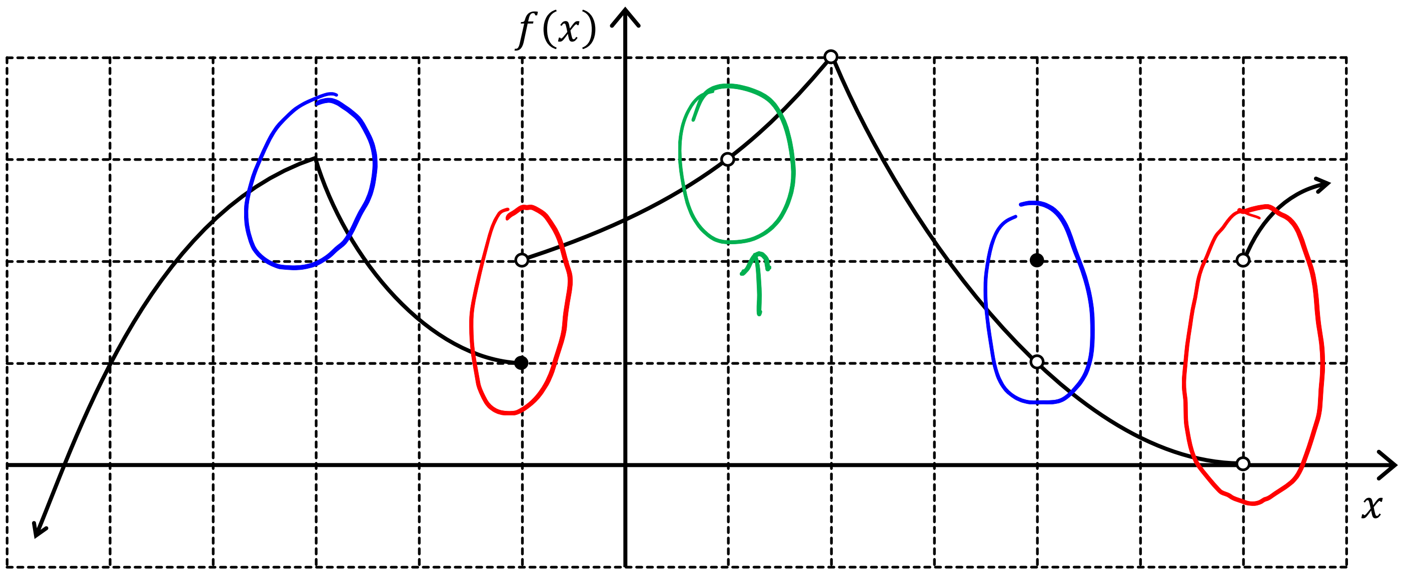
Holes, jumps, and points in the wrong places are easy to spot on a graph. A reasonable question, though, is the following:

> *Is there some way of analyzing the formula for a function to determine if its graph would have any holes & jumps?*

The answer is, *yes*. The concept of *continuity* allows one to analyze the formula for a function and determine if (and where) its graph would have holes & jumps.

Before, we see the *definition of continuity*, let's look again at the graph from the previous video, and review how the accompanying table of questions was filled out, because we will gain some insight into how *continuity* will be defined.

Questions from Video for H02: Limits for a Function Given by a Graph (Section 2.1)



x-value	limit from left	limit from right	limit	y-value
-3	$\lim_{x\to -3^-} f(x) =$ 3	$\lim_{x\to -3^+} f(x) =$ 3	$\lim_{x\to -3} f(x) =$ 3	$f(-3) =$ 3
-1	$\lim_{x\to -1^-} f(x) =$ 1	$\lim_{x\to -1^+} f(x) =$ 2	$\lim_{x\to -1} f(x) =$ DNE	$f(-1) =$ 1
1	$\lim_{x\to 1^-} f(x) =$ 3	$\lim_{x\to 1^+} f(x) =$ 3	$\lim_{x\to 1} f(x) =$ 3	$f(1) =$ DNE
4	$\lim_{x\to 4^-} f(x) =$ 1	$\lim_{x\to 4^+} f(x) =$ 1	$\lim_{x\to 4} f(x) =$ 1	$f(4) =$ 2
6	$\lim_{x\to 6^-} f(x) =$ 0	$\lim_{x\to 6^+} f(x) =$ 2	$\lim_{x\to 6} f(x) =$ DNE	$f(6) =$ DNE

Observations:

At $x = -3$, there is no break or jump or point in the wrong place, and at $x = -3$ the following three things are all true:

1) $\lim_{x \to -3} f(x)$ exists
2) $f(-3)$ exists
3) The number that is the limit equals the number that is the y value. That is, $\lim_{x \to -3} f(x) = f(-3)$

Observations, continued: At all the x values where there is a break or a jump, one or more of those three things is not true.

At $x = -1$, there is a jump in the graph

1) $\lim_{x \to -1} f(x)$ does not exist

2) $f(-1)$ exists

3) The limit and the y value can't be equal, because there is no limit.

At $x = 1$, there is a hole in the graph

1) The limit exists: $\lim_{x \to 1} f(x) = 3$

2) $f(1)$ does not exist

3) The limit and the y value can't be equal, because there is no y value.

At $x = 4$, there is a hole in the graph and a dot in the wrong place

1) The limit exists: $\lim_{x \to 4} f(x) = 1$

2) $f(4) = \cancel{3}$ 2

3) The limit and the y value are not equal equal

After making the above observations about the graph, we are in a good position to understand the following *definition of continuity*.

Definition of *Continuity at a particular x value*

Words: *The function f is continuous at $x = c$.*

Meaning: the function f passes these three tests:

Test 1: $\lim_{x \to c} f(x)$ exists

Test 1a: $\lim_{x \to c^-} f(x)$ exists

Test 1b: $\lim_{x \to c^+} f(x)$ exists

Test 1c: The numbers in test 1a and 1b agree.

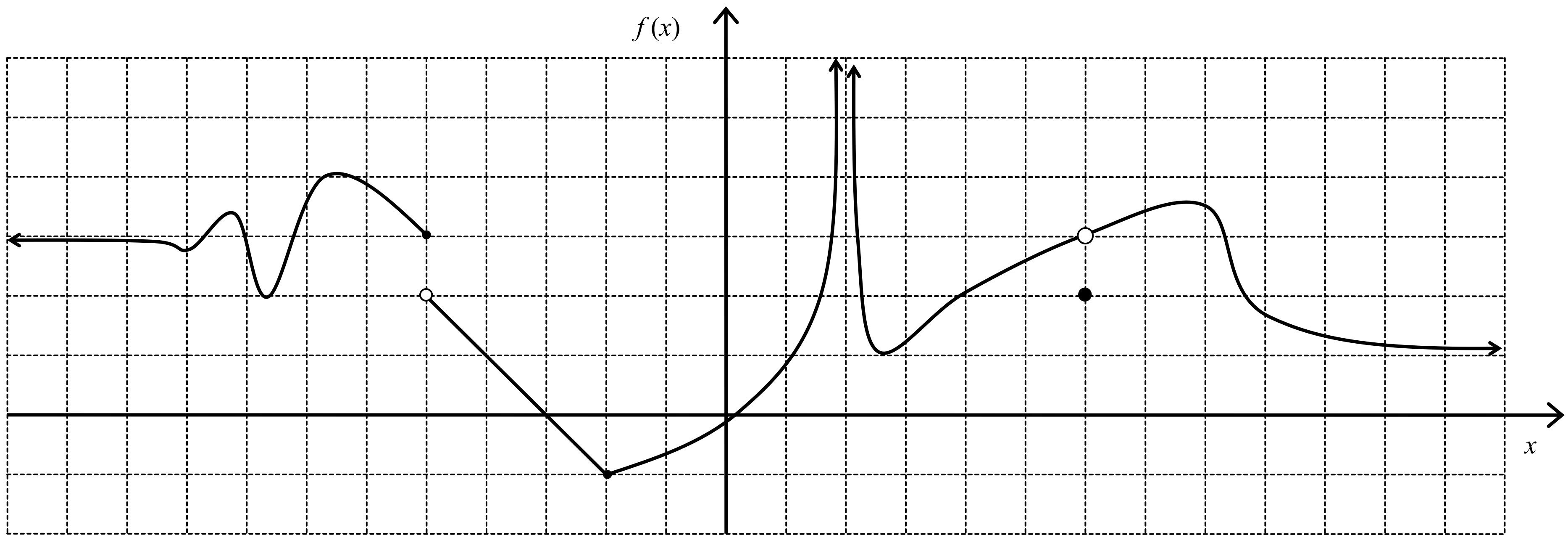
Test 2: $f(c)$ exists

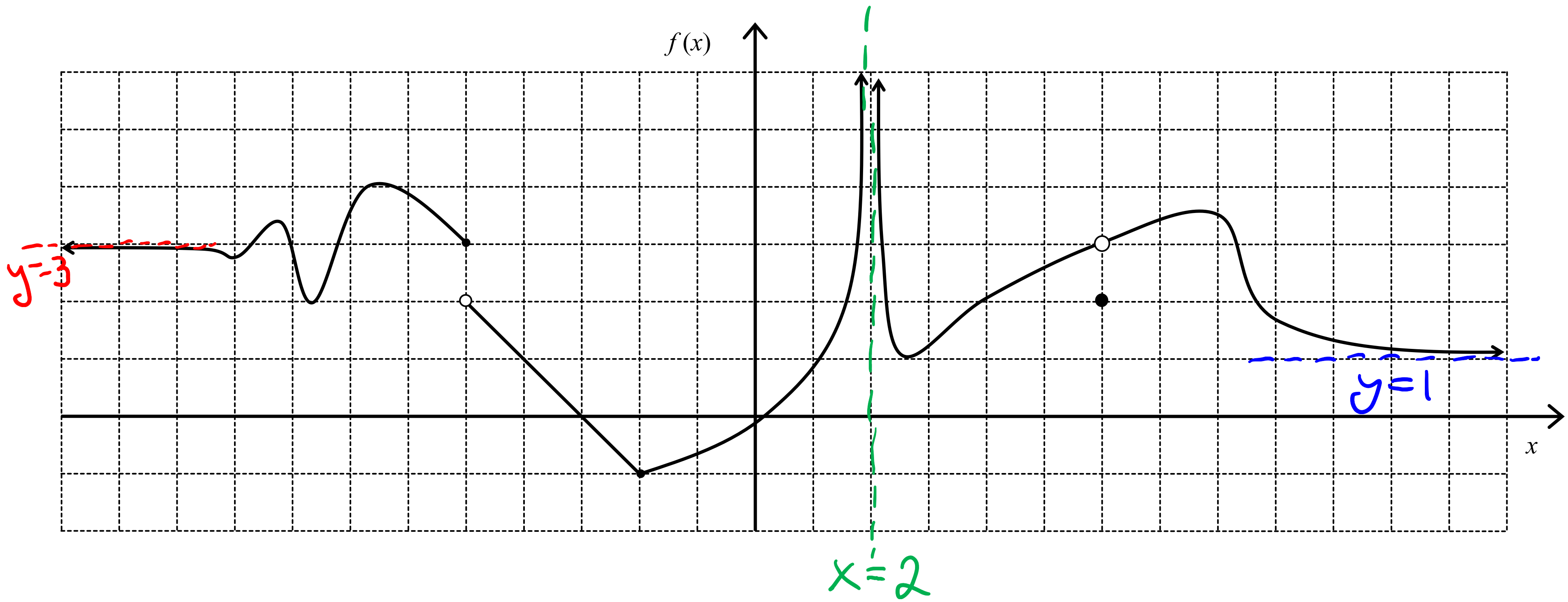
Test 3: The numbers in *Test 1* and *Test 2* agree.

[Example]

In this example, you are asked various questions about a graph. Some of the questions are about *Continuity* (our new concept, from Section 2.3), but some of the questions are about *Limits* and *Limits involving Infinity* (old concepts from Sections 2.1 and 2.2)

Here is the graph:



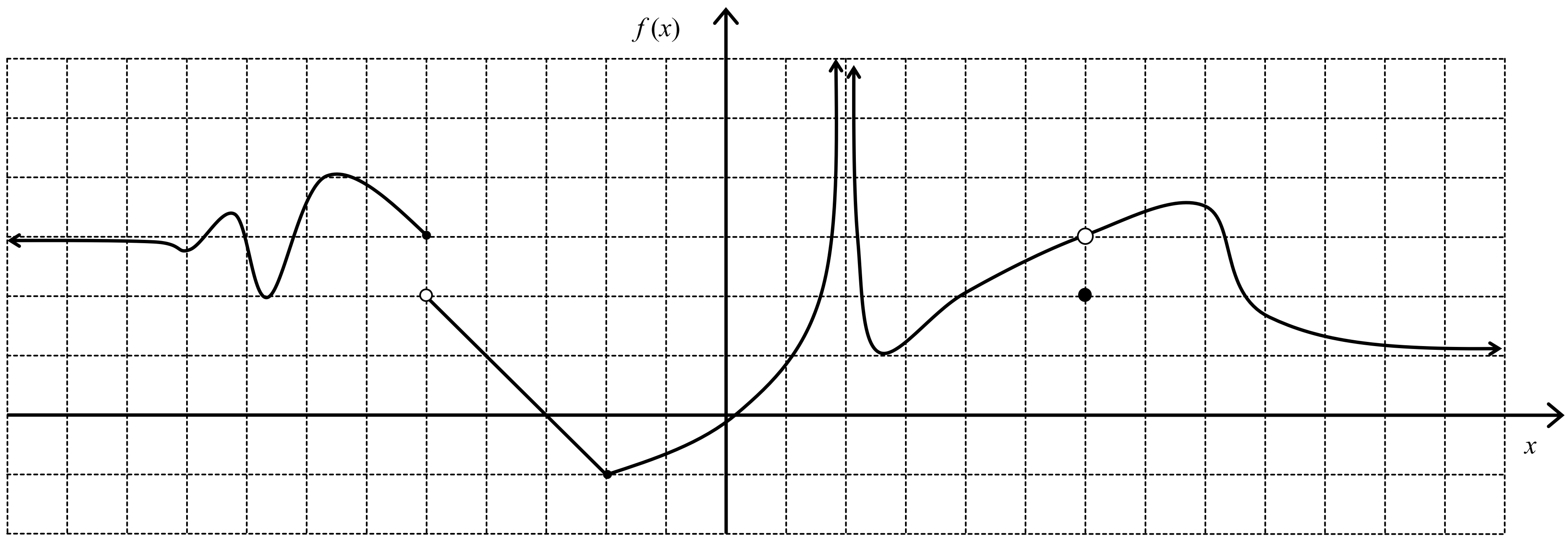


(1) For each asymptote, give the line equation and say whether it is horizontal or vertical.

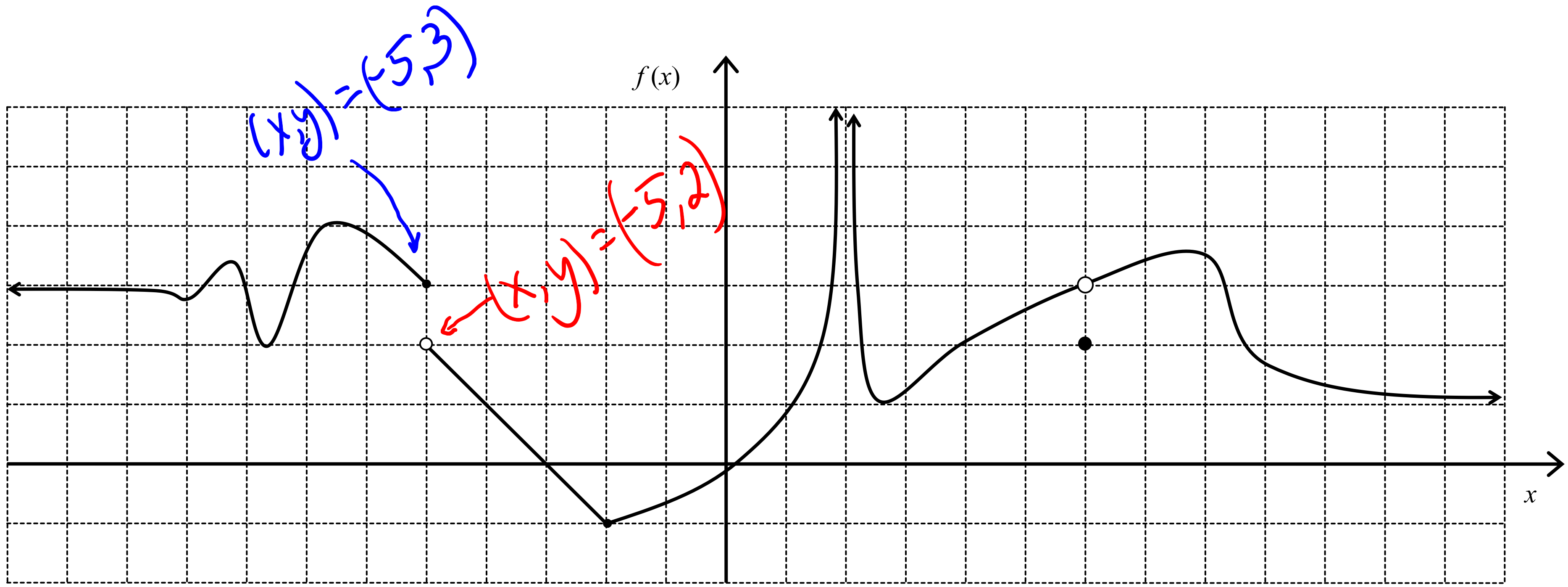
Horizontal asymptote on the left with line equation $y=3$

Vertical asymptote with line equation $x=2$

Horizontal asymptote on the right with line equation $y=1$



(2) $\lim_{x \to -\infty} f(x) =$ 3 because horizontal asymptote on the left with line equation $y = 3$

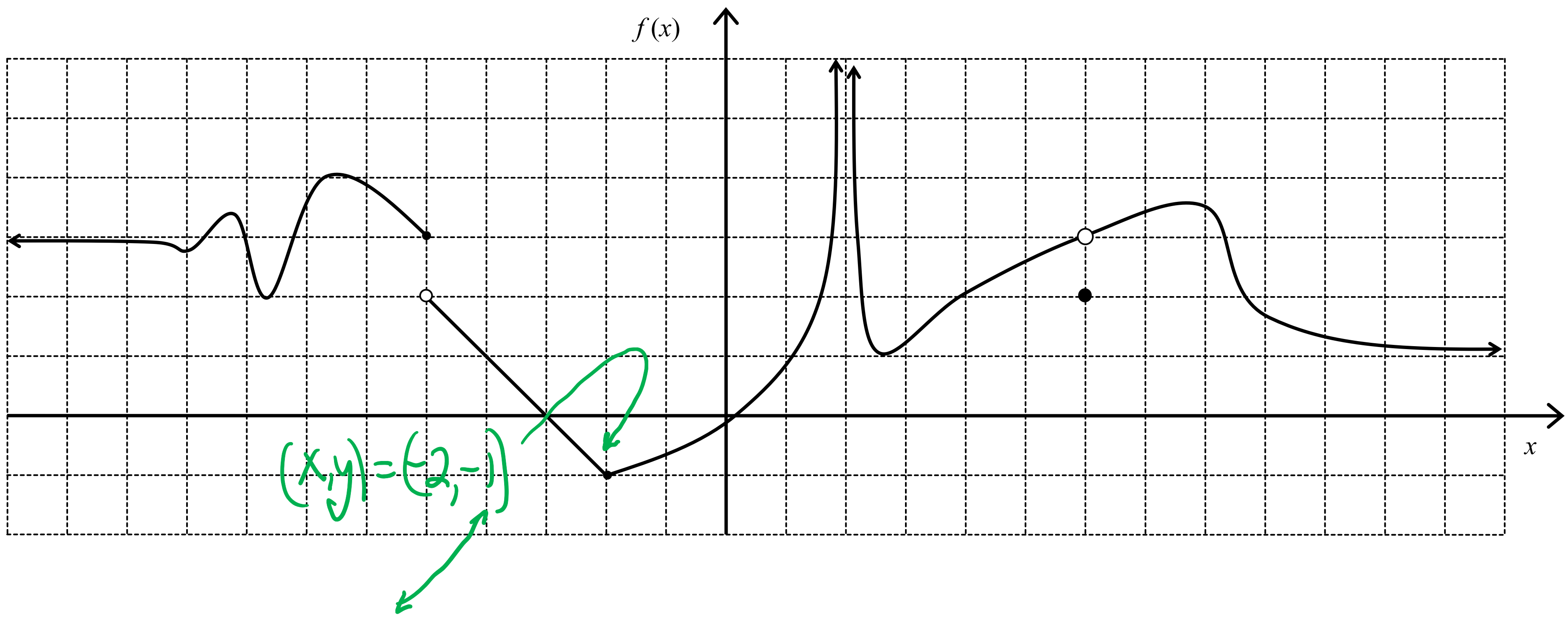


(3) $\lim_{x \to -5} f(x) =$

left limit $\lim_{x \to -5^-} f(x) = 3$ because the graph is approaching the location $(x,y) = (-5,3)$ from the left.

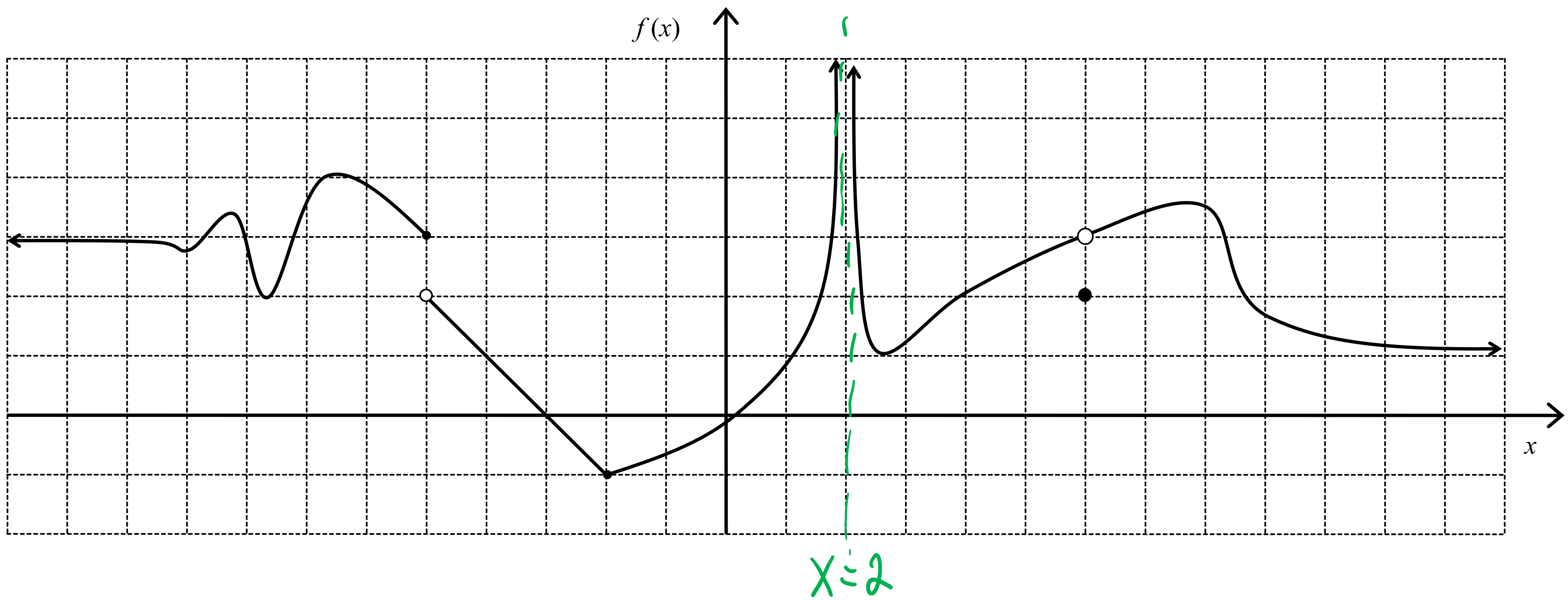
right limit $\lim_{x \to -5^+} f(x) = 2$ because the graph is approaching the location $(x,y) = (-5,2)$ from the right.

$\lim_{x \to -5} f(x)$ Does Not Exist because left + right limits do not match

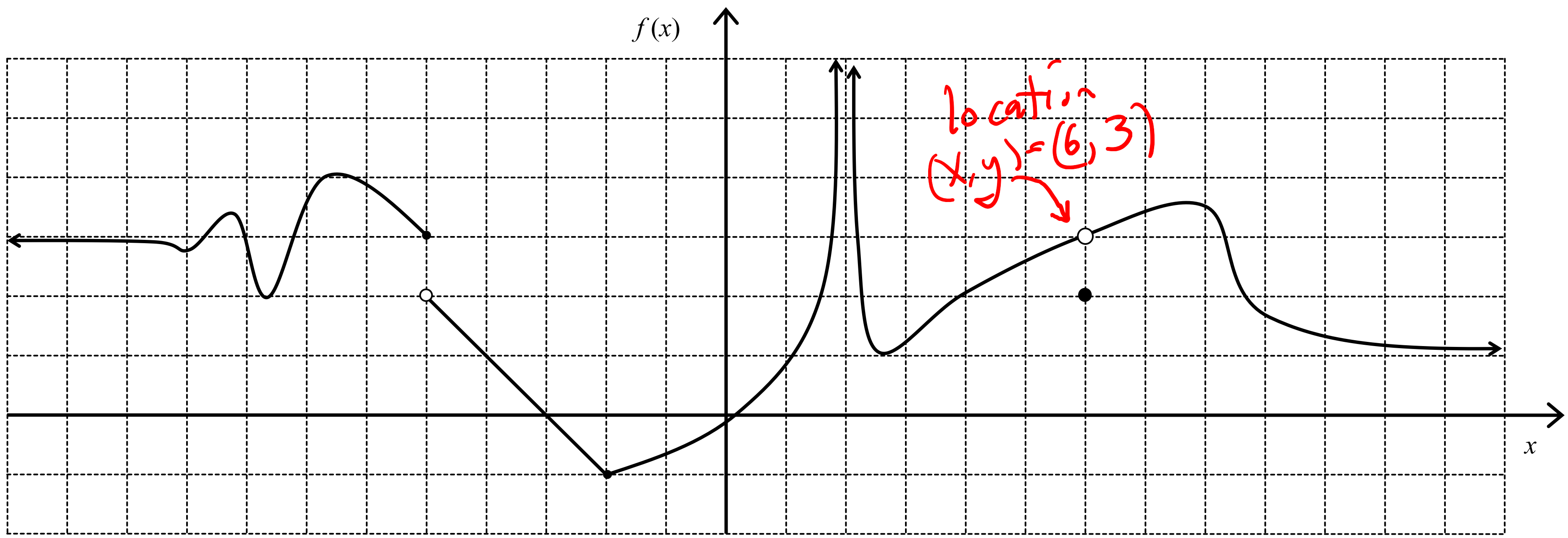


(4) $\lim_{x \to -2} f(x) = -1$

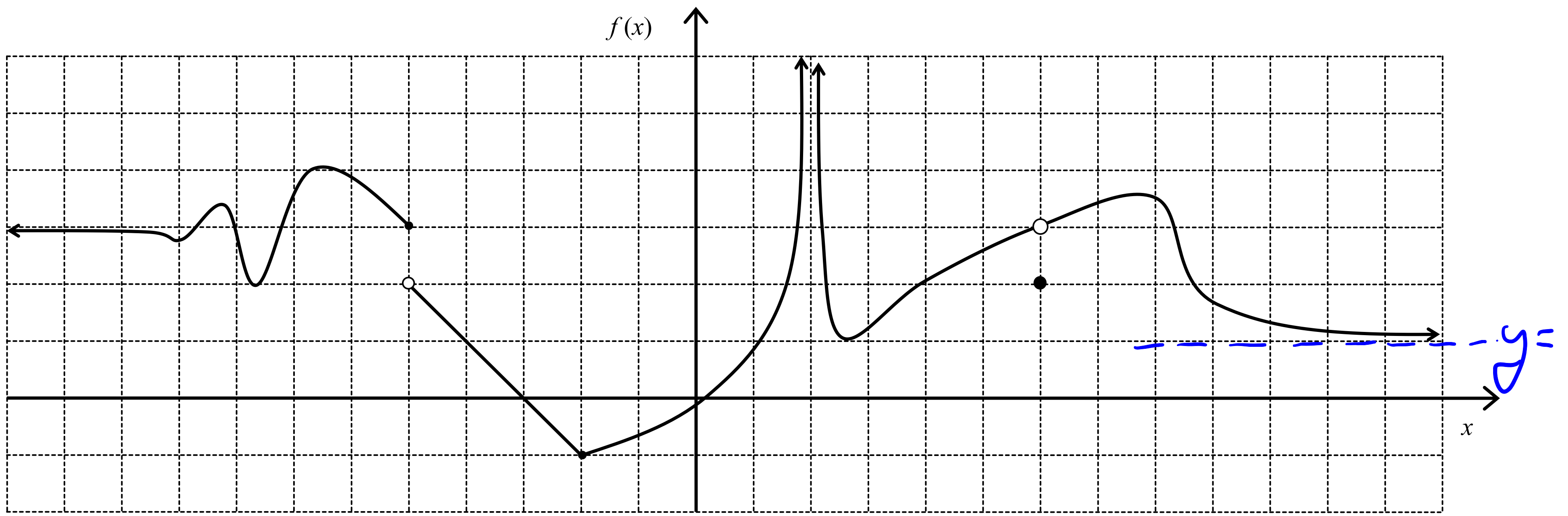
because graph is heading for the location $(x,y)=(-2,-1)$



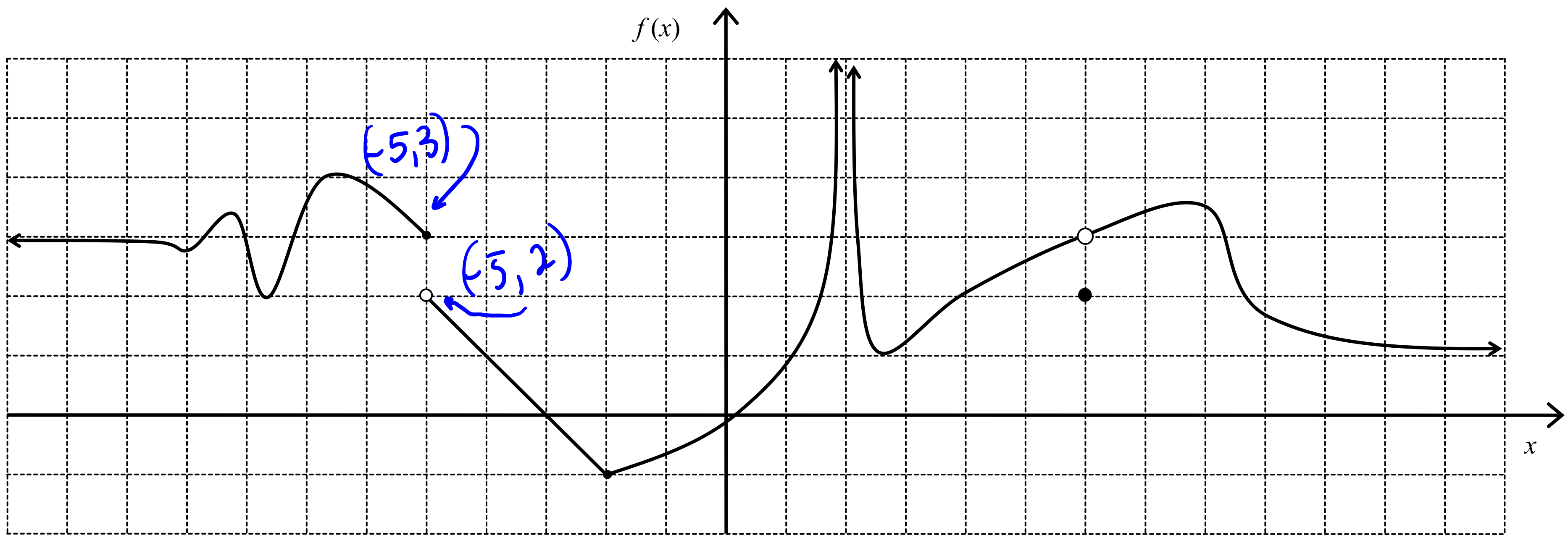
(5) $\lim_{x \to 2} f(x) = \infty$ because there is a vertical asymptote with line equation X=2 and the graph goes up along both sides of that asymptote.



(6) $\lim_{x\to 6} f(x) = 3$ because the graph is heading for the location $(x,y)=(6,3)$



(7) $\lim_{x\to\infty} f(x) =$ 1 because the graph has a horizontal asymptote on the right with line equation $y=1$



(8) Is f continuous at $a = -5$? If not, explain why not.

Consider the continuity test

Test 1a $\lim_{x \to -5^-} f(x) = 3$

Test 1b $\lim_{x \to -5^+} f(x) = 2$

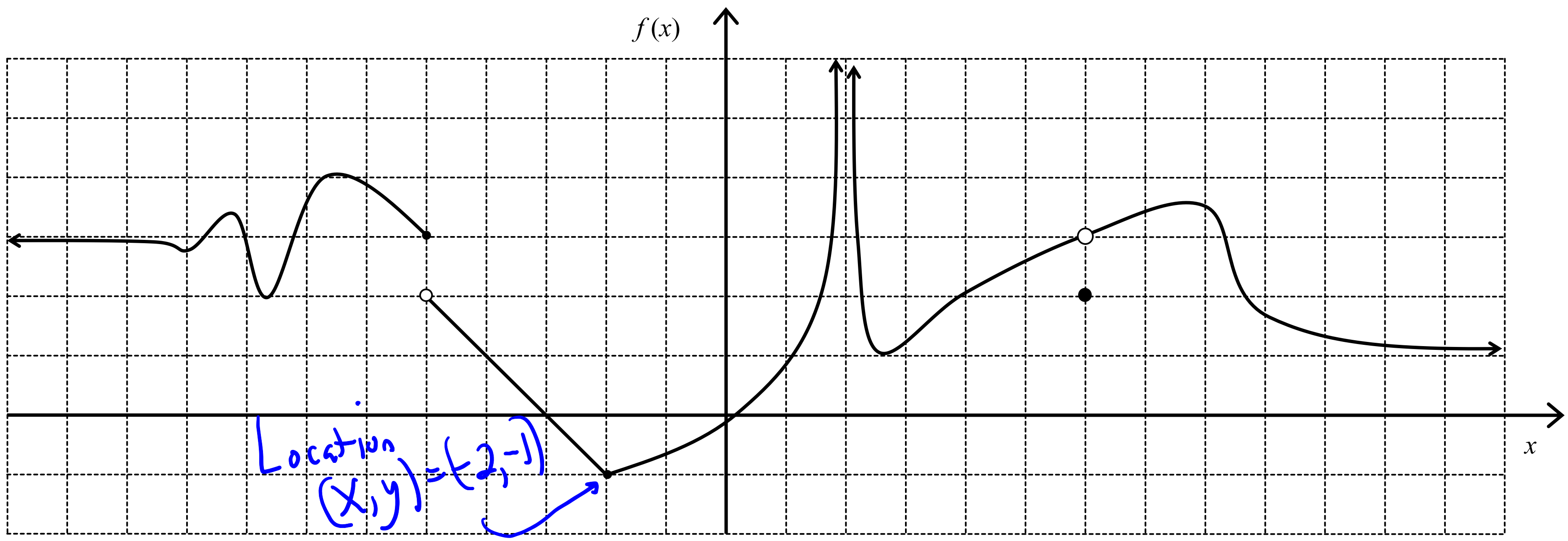
Test 1c The left + right limits do not match

Fails test 1c the limit doesn't exist

Test 2 $f(-5) = 3$ because point at $(x,y) = (-5,3)$

Test 3 fails test 3 because the limit does not exist.

Conclusion: the function is not continuous at $x = -5$



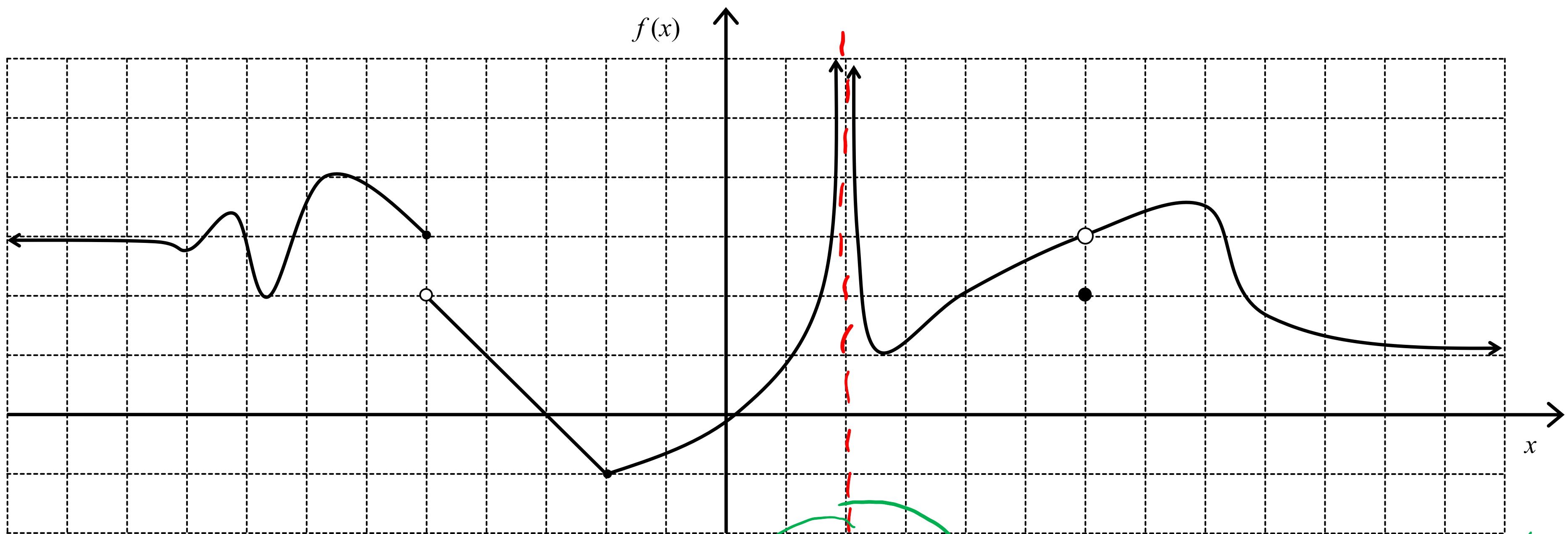
(9) Is f continuous at $a = -2$? If not, explain why not.

Test 1 does limit exist? $\lim_{x \to -2} f(x) = -1$ because graph is heading for the location $(x,y) = (-2,-1)$

Test 2 does y-value exist? $f(-2) = -1$ because there is a point on the graph at $(x,y) = (-2,-1)$

Test 3 $\lim_{x \to -2} f(x) = -1 = f(-2)$ results of Test 1 and Test 2 match.

Conclusion: the function is continuous at $x = -2$.



There was a mistake in the video here. I spoke and wrote $x = -2$.

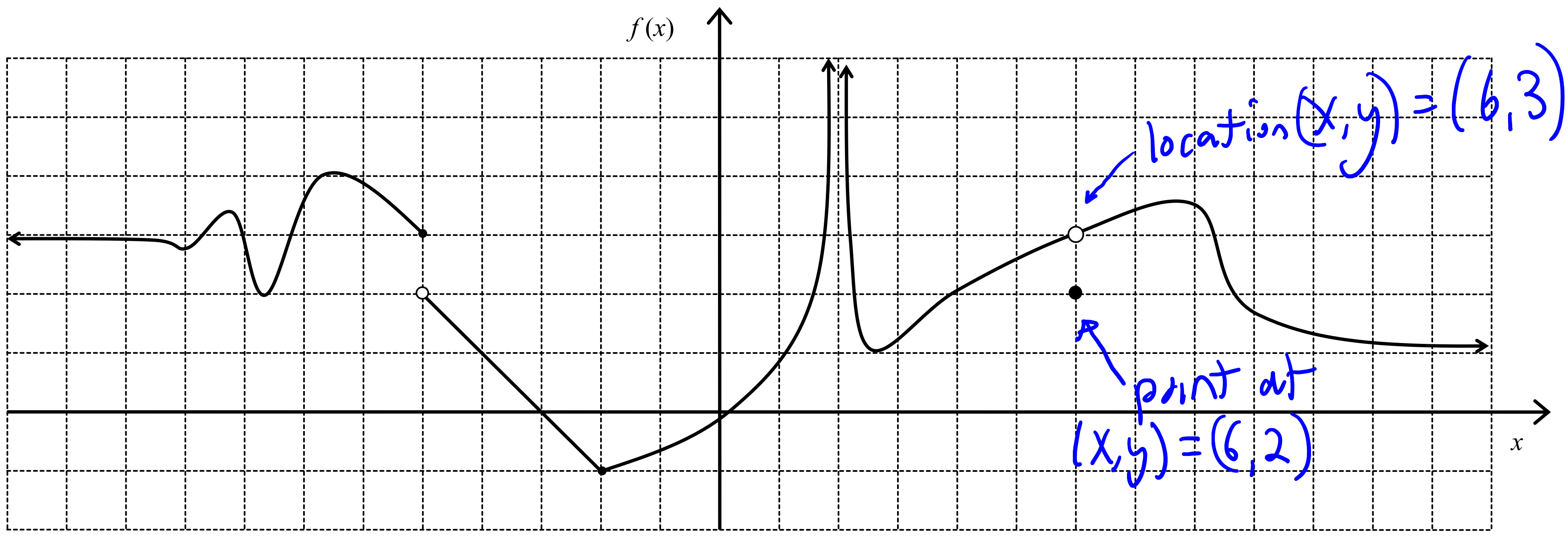
(10) Is f continuous at $a = 2$? If not, explain why not.

Test 1 $\lim_{x \to 2} f(x) = \infty$

Test 2 $f(2)$ DNE because there is no point on the graph at $x = 2$

Test 3 fails test 3 because results of test 1 and test 2 are not equal.

Conclude: The function is not continuous at $x = 2$



(11) Is f continuous at $a = 6$? If not, explain why not.

Test 1 $\lim_{x\to 6} f(x) = 3$ because graph is heading for the location $(x,y) = (6,3)$

Test 2 $f(6) = 2$ because point on graph at $(x,y) = (6,2)$

Test 3 $\lim_{x\to 6} f(x) \neq f(6)$ so fails test 3

Conclusion: f is not continuous at $x=6$ because it Fails test 3.

End of [Example]

End of Video