Section 2.4 Rates of Change

Subject for this video: Prerequisite Skills: Building and Simplifying Expressions

Reading: Section 1.1, page 10, including Example 6

Homework: H24: Prerequisite Skills: Building and Simplifying Expressions (1.1#61,63,73,78)

[Example] Let $f(x) = -3x^2 + 15x - 7$

Find the following quantities. In question $[F]$, assume that $h \neq 0$.

$[\boldsymbol{A}]$ $f(4x)$

$[\boldsymbol{B}]$ $f(-4)$

$[\boldsymbol{C}]$ $f(4)$

$[\boldsymbol{D}]$ $f(4+h)$

$[\boldsymbol{D}]$ $f(4+h) - f(4)$

$[\boldsymbol{F}]$ $\dfrac{f(4+h) - f(4)}{h}$

Solution to (A) $f(x) = -3x^2 + 15x - 7$

we need $f(4x)$

Build the empty version of f

$f(\ ) = -3(\ )^2 + 15(\ ) - 7$ empty version.

Now put $4x$ into each parentheses

$$f(4x) = -3(4x)^2 + 15(4x) - 7$$

$$= -3(16x^2) + 60x - 7$$

$$= -48x^2 + 60x - 7$$

Solution to [B] Find $f(-4)$

Return to the empty version

$$f(\ ) = -3(\ )^2 + 15(\ ) - 7$$

$$f(-4) = -3(-4)^2 + 15(-4) - 7$$

$$= -3(16) - 60 - 7$$

$$= -48 - 60 - 7$$

$$= -115$$

Solution to [c] We need $f(4)$.

Start with empty version

$$f(\ ) = -3(\ \ )^2 + 15(\ \ ) - 7$$

Substitute in the number 4

$$f(4) = -3(4)^2 + 15(4) - 7$$

$$= -3 \cdot 16 + 60 - 7$$

$$= -48 + 60 - 7$$

$$= 12 - 7$$

$$= \boxed{5}$$

Solution to [D] We need $f(4+h)$

Start with the empty version

$$f(\quad) = -3(\quad)^2 + 15(\quad) - 7$$

Substitute $4+h$ into each parentheses

$$f(4+h) = -3\left((4+h)^2\right) + 15(4+h) - 7$$

$$= -3\left(16+8h+h^2\right) + 60 + 15h - 7$$

$$= -48 - 24h - 3h^2 + 60 + 15h - 7$$

$$= (-48+60-7) + (-24h+15h) - 3h^2$$

$$= 5 - 9h - 3h^2$$

$(4+h)^2 = (4+h)(4+h) = 16+4h+4h+h^2 = 16+8h+h^2$

Solution to [E] we need to build $f(4+h) - f(4)$

$$f(4+h) - f(4) = \left(\cancel{5} - 9h - 3h^2\right) - \left(\cancel{5}\right)$$

from [D] from [C]

$$= -9h - 3h^2$$

Solution to [F] We need to build the ratio

$$\frac{f(4+h)-f(4)}{h} \quad \text{(assuming } h \neq 0\text{)}$$

Remark: This is a difference quotient

That is, it is of the form $\frac{\Delta y}{\Delta x}$

Notice that we found the numerator in part (E)

$$\frac{f(4+h)-f(4)}{h} = \frac{-9h-3h^2}{h} = \frac{h(-9-3h)}{h} = -9-3h$$

use result from (E)

factor out h in numerator

Since $h \neq 0$ we can cancel $\frac{h}{h}$

End of Video