Subject for this video: The Constant Function Rule and the Power Rule

Reading:

- **General:** Section 2.5, Basic Differentiation Properties
- **More specifically:** middle of p. 145 – middle of p. 148, Examples 1,2,3

Homework:

Constant Function Rule and Power Rule (2.5 #9,11,13,15,17,19)

Today we will learn our first derivative rules. Before embarking on that, it is useful to review two things.

- One concept is from the prerequisite material: *power functions*.
- The other concept is from the last two videos: the *Definition of the Derivative* and the *Four Step Process*.

Review of Power Functions

Recall that a power function is a function of the form $f(x) = x^k$, where k is a real number.

Forms of Power Functions

Of course, power functions can be presented in various forms. It is useful to have names for the forms when working with them.

Power function form is the basic form x^k. Note that k can be positive or negative, or zero.
For instance, these are all power functions in power function form:

$$x^7, x^1, x^0, x^{-5}$$

People are often confused by expressions with negative exponents. Because of that, it is often a good idea to present functions in a form that has only positive exponents. We will say that a power function is in ***positive exponent form*** if all of its exponents are positive. The rule of exponents $x^{-k} = \frac{1}{x^k}$ allows us to convert between power function form and positive exponent form. So, for instance, $x^{-5} = \frac{1}{x^5}$

Most students are comfortable with the square root function, $f(x) = \sqrt{x}$. But

power functions presented in more complicated ***radical form*** such as $g(x) = \sqrt[5]{x^3}$ can lead to confusion. Furthermore, it is not possible to do much arithmetic with power functions in radical form. One must convert them to power function form. To do this, one must recall the definition of what the radical form means:

$$\sqrt[n]{x^m} = x^{m/n}$$

Rational exponent form is just what it sounds like: a power function with an exponent that is expressed as a rational number. For instance, here are two power functions in rational exponent form:

$$x^{-7/3}, \frac{1}{x^{2/5}}$$

Working with Rational Exponents

Two important basic facts will be useful in working with power functions that have rational exponents. The first useful basic fact is the commutative law of multiplication

$$\frac{m}{n} = m \cdot \frac{1}{n} = \frac{1}{n} \cdot m$$

The second useful basic fact is the rule of exponents

$$a^{b \cdot c} = (a^b)^c$$

These two basic facts can be combined to give us insight into how to work with rational exponents

- We can do this $x^{m/n} = x^{m \cdot (1/n)} = (x^m)^{1/n}$
- But can also do this $x^{m/n} = x^{(1/n) \cdot m} = \left(x^{1/n}\right)^m$

Both of the lines above are true, but the second one will make calculations easier. For example

- We can do this $8^{2/3} = 8^{2 \cdot (1/3)} = (8^2)^{1/3} = (64)^{1/3} = 4$
- But can also do this $8^{2/3} = 8^{(1/3) \cdot 2} = \left(8^{1/3}\right)^2 = (2)^2 = 4$

Both lines give the same result, but the second line is easier.

Review of the Definition of the Derivative and the Four Step Process:

Recall the Definition of the Derivative from Section 2.4 of the book and recent videos:

Definition of the *Derivative*

Symbol: $f'(x)$

Spoken: f *prime of* x

Also spoken: *the derivative of* f *of* x

Meaning: the function $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

Used For: When an actual number $x = a$ is substituted into the derivative function $f'(x)$, the resulting number $f'(a)$ can be interpreted as

- the slope of the line tangent to the graph of $f(x)$ at $x = a$
- the slope of the the graph of $f(x)$ at $x = a$
- the instantaneous rate of change of $f(x)$ at $x = a$
- If $f(x)$ is a position function for a moving object, the number $f'(a)$ is the velocity of the object at time $x = a$.

And recall the outline of a process for computing the derivative:

Four Step Process for finding $f'(x)$ using the *Definition of the Derivative*

Present the goal: to find $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

Get parts:

Step 1: Find the expression $f(x+h)$

Step 2: Find the expression $f(x+h) - f(x)$

Step 3: Find the expression $\frac{f(x+h)-f(x)}{h}$

Find the limit:

Step 4: Build the expression $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ and compute the limit.

Explain clearly:

- Point out limits that are indeterminate forms.
- If cancellation is done, explain clearly why it is justified.

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Finding derivatives using the *Definition of the Derivative* (with your work organized by the *Four Step Process*) is very hard.

In the current Section 2.5, we *will not* use the Definition of the Derivative. We will use the so-called *Derivative Rules*. These are just certain forms of derivatives that occur frequently, and for which a certain *pattern* in the result can be observed and put into the form of a *rule*.

Before learning some derivative rules, we need to learn about some new notation for the derivative.

New Notation for the Derivative

New symbol: $\frac{df(x)}{dx}$

Spoken: *dee* f *of* x *dee* x

Another new symbol: $\frac{d}{dx}f(x)$

Spoken: *dee over dee* x *of* f *of* x **Also spoken:** *dee dee* x *of* f *of* x

Meaning in words: The derivative of the function $f(x)$

Meaning in symbols: $f'(x)$

Another new symbol: $\frac{dy}{dx}$

Spoken: *dee* y *dee* x

Usage: The letter y represents a function of x that has been introduced by some equation.

That is, using function notation, the function would be denoted $y(x)$

Meaning in words: The derivative of the function $y(x)$

Meaning in symbols: $y'(x)$

In the examples in this video, and throughout the book, you will see all this notation used.

The new symbol $\frac{df(x)}{dx}$ might look kind of random to you, but there is a certain reason for it. Remember that the definition of the derivative is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The expression inside the limit is a *difference quotient*, computing the slope of a secant line that touches the graph of $f(x)$ at the two points $\big(x, f(x)\big)$ and $\big(x+h, f(x+h)\big)$. That is,

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

The symbol Δ is the capital Greek letter Delta. In these symbols, Δ denotes the *change* in a quantity. The h in the denominator is the Δx, the *change in* x. The limit as $h \to 0$ could also be thought of as the limit as $\Delta x \to 0$. So, the derivative could be thought of as

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \quad or \quad f'(x) = \lim_{\Delta x \to 0} \frac{\Delta f(x)}{\Delta x}$$

The new symbol $\frac{df(x)}{dx}$ with its small letter d is meant to signify that it is the result of a *limit* of the symbol $\frac{\Delta f(x)}{\Delta x}$ with its capital letter $Delta$. That is,

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f(x)}{\Delta x}$$

Finally, we get to the Derivative Rule.

We will discuss two today: The first is the *Constant Function Rule*.

The Constant Function Rule

This rule is used for finding the derivative of a *constant* function.

Two equation form: If $f(x) = c$ then $f'(x) = 0$.

Single equation form: $\frac{d}{dx}c = 0$

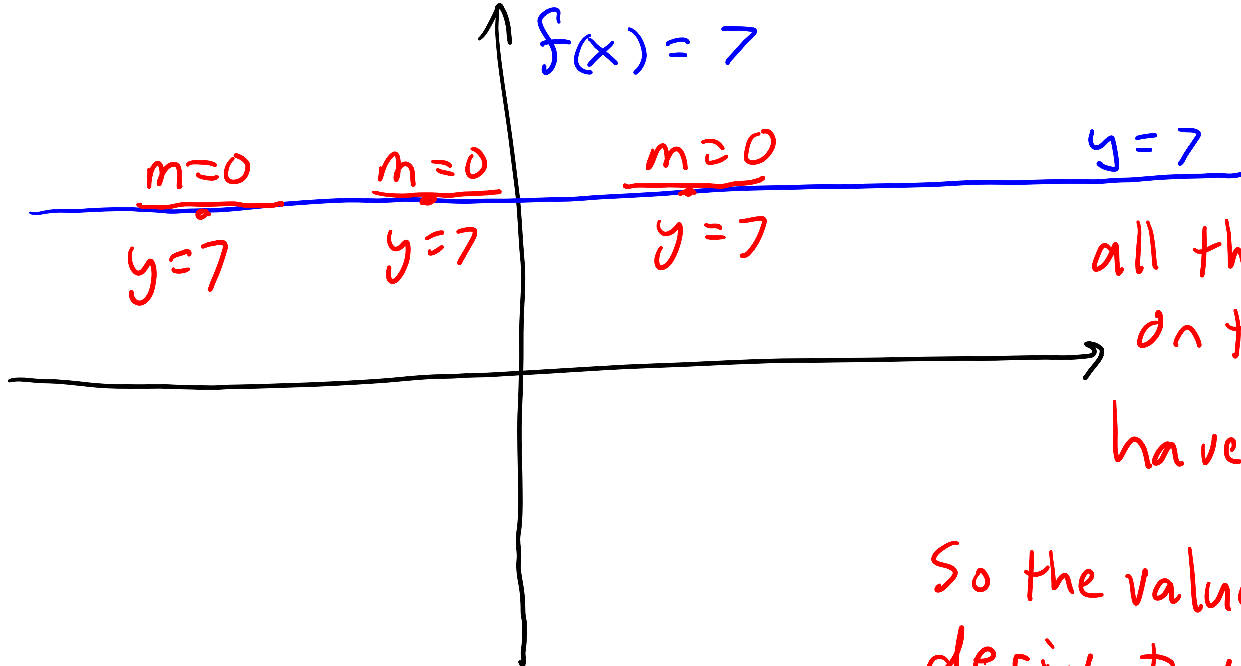
[Example 1] (A) Let $f(x) = 7$. Find $f'(x)$

Solution: Using Two Equation form

$f(x) = 7 =$ constant, so $f'(x) = 0$

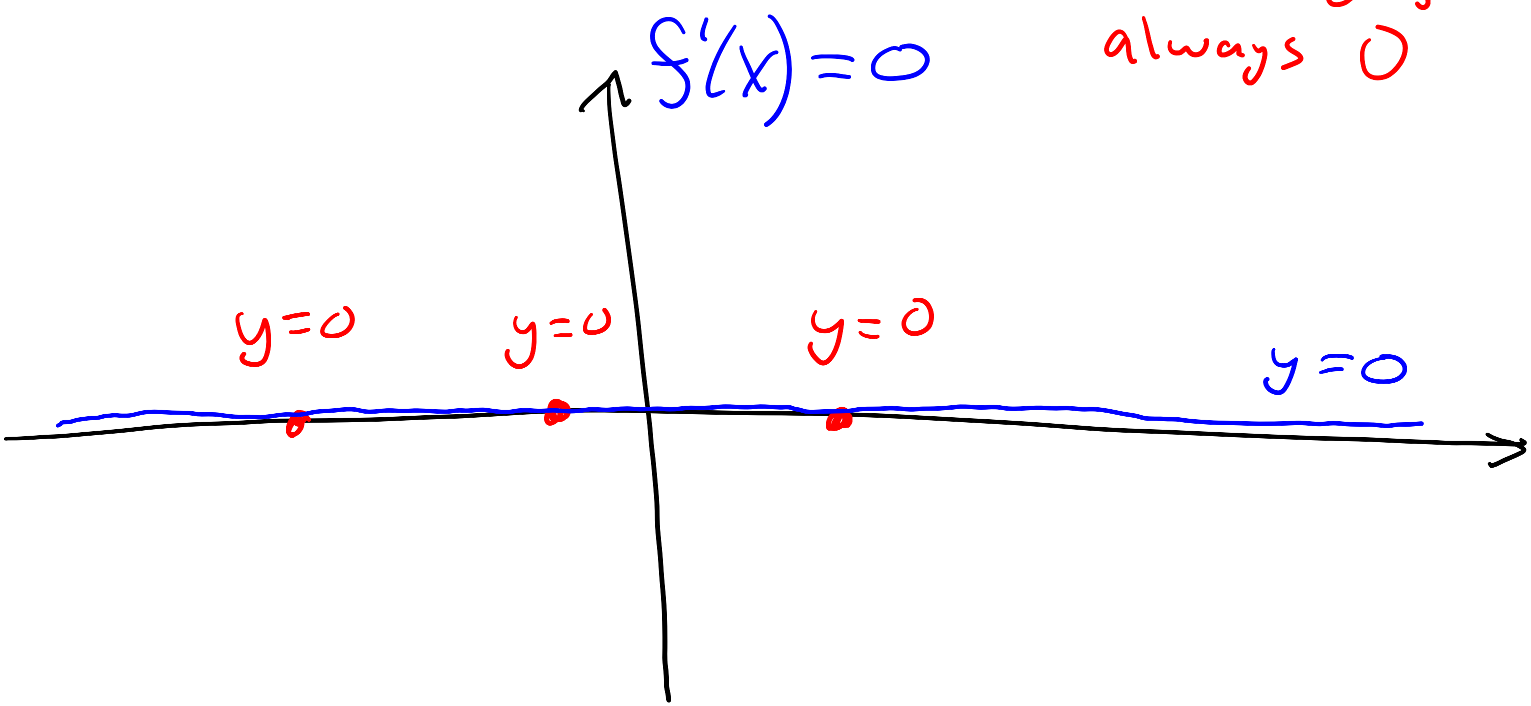
Single equation form $\frac{d}{dx}7 = 0$

(B) Using a graph, discuss why the result makes sense.



all the tangent lines on the graph of f(x) have slope m=0

So the value of the derivative is always 0



The second derivative rule that we will discuss two today is the *Power Rule*.

Recall that a power function is a function of the form $f(x) = x^k$, where k is a real number.

The Power Rule

This rule is used for finding the derivative of a *power* function.

Two equation form: If $f(x) = x^n$ then $f'(x) = nx^{n-1}$.

Single equation form: $\frac{d}{dx}x^n = nx^{n-1}$

Note that in both presentations of the *Power Rule*, the original power function (before finding the derivative) is in the form x^n. That is, the function is in *power function form*.

[Example 2] Let $f(x) = x^7$. Find $f'(x)$

nx^{n-1}

Solution

Use two equation form of the Power Rule

Since $f(x) = x^{7} \leftarrow n=7$, we know $f'(x) = \underbrace{7 \cdot x^{7-1}} = 7x^{6}$

identify power n.

Use single equation form of the power rule

$$\underbrace{\frac{d}{dx} x^{7}}_{\text{left side of power rule}} \leftarrow n=7 \quad = \underbrace{7 \cdot x^{7-1}}_{\text{right side of power rule}} = 7x^{6}$$

[Example 3] Find $\frac{dy}{dx}$ for $y = \frac{1}{x^7}$

original function is in positive exponent form

Solution

Rewrite our function in power function form

$$y = \frac{1}{x^7} = x^{-7}$$

Now find the derivative

$n = -7$

$$\underbrace{\frac{dy}{dx} = \frac{d}{dx}x^{-7}}_{\text{left side of power rule}} = \underbrace{-7 \cdot x^{-7-1}}_{\text{right side of power rule}} = \underbrace{-7x^{-8}}_{\text{power function form}} = -7 \cdot \frac{1}{x^8} = \underbrace{\frac{-7}{x^8}}_{\text{positive exponent form}}$$

[Example 4] Let $f(x) = x^{1/3}$.

(A) Find $f'(x)$

(B) Find $f(8)$ and $f'(8)$

Solution

$$\text{(A)}\quad f'(x) = \underbrace{\frac{d}{dx}x^{1/3}}_{\text{left side of power rule}} \xleftarrow{\ n=\frac{1}{3}\ } \quad = \underbrace{\frac{1}{3}\cdot x^{\frac{1}{3}-1}}_{\text{right side of power rule}} = \underbrace{\frac{1}{3}x^{-\frac{2}{3}}}_{\text{power function form}} = \frac{1}{3}\cdot\frac{1}{x^{2/3}} = \underbrace{\frac{1}{3x^{2/3}}}_{\text{positive exponent form}}$$

three steps of rewriting the result.

$$\text{(B)}\quad f(8) = 8^{1/3} = 2$$

$$f'(8) = \frac{1}{3(8^{2/3})} = \frac{1}{3\left(\left(8\right)^{1/3}\right)^2} = \frac{1}{3\cdot\left(\left(2\right)^2\right)} = \frac{1}{3\cdot 4} = \frac{1}{12}$$

[Example 5] Find $\frac{d}{dx}\sqrt{x}$

Solution Rewrite function in power function form

$$\sqrt{x} = x^{1/2}$$

Then take the derivative

$$\underbrace{\frac{d}{dx} x^{1/2}}_{\text{left side of power rule}} \overset{\leftarrow\, n = 1/2}{=} \underbrace{\frac{1}{2} \cdot x^{\frac{1}{2}-1}}_{\text{right side of power rule}} \underbrace{= \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{1/2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}}_{\text{Four steps of rewriting the result.}}$$

[Example 6] Find $\frac{dy}{dx}$ for $y = \frac{1}{x^{3/5}}$

Solution Start by rewriting the function in power function form

$$\frac{1}{x^{3/5}} = x^{-3/5}$$

Now find the derivative

$$\frac{dy}{dx} = \underbrace{\frac{d}{dx} x^{-3/5}}_{\text{left side of power rule}} \quad n = \frac{-3}{5}$$

$$= \underbrace{\frac{-3}{5} \cdot x^{-\frac{3}{5}-1}}_{\text{right side of power rule}} \underbrace{= \frac{-3}{5} \cdot x^{-\frac{8}{5}} = \frac{-3}{5} \cdot \frac{1}{x^{8/5}} = \frac{-3}{5x^{8/5}}}_{\text{three steps of rewriting the result.}}$$

power function form

Positive exponent form

[Example 7] (A) Find the derivative of $f(x) = x$

Solution Rewrite the function in power function form

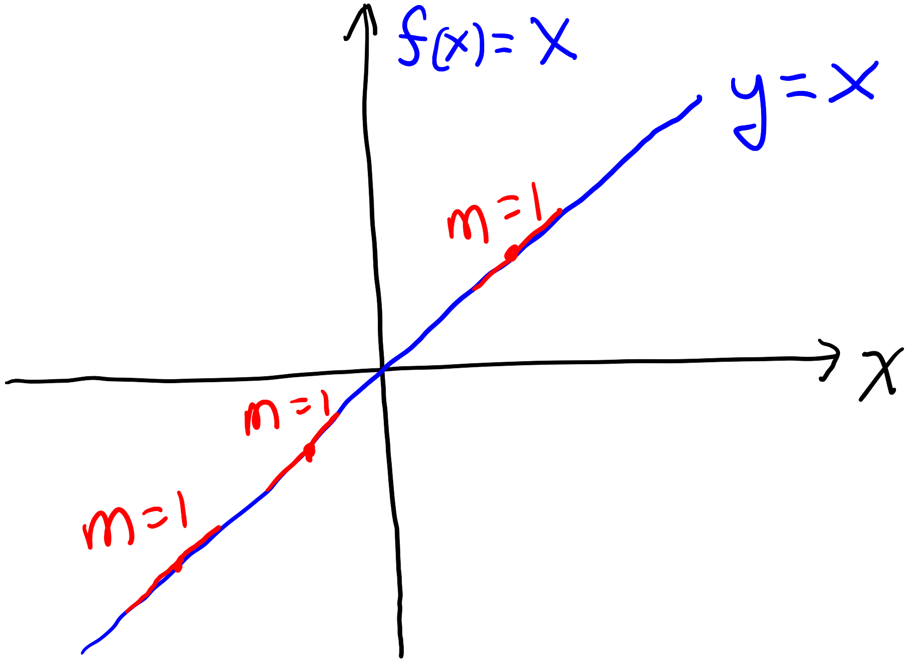
$$f(x) = x = x^1$$

Now find the derivative

$\leftarrow n=1$

$$\underbrace{\frac{d}{dx}}_{\text{left side of power rule}} x^1 = \underbrace{1 \cdot x^{1-1}}_{\text{right side of power rule}} \underbrace{= 1 \cdot x^0 = 1 \cdot 1 = 1}_{\text{three steps of simplifying the result.}}$$

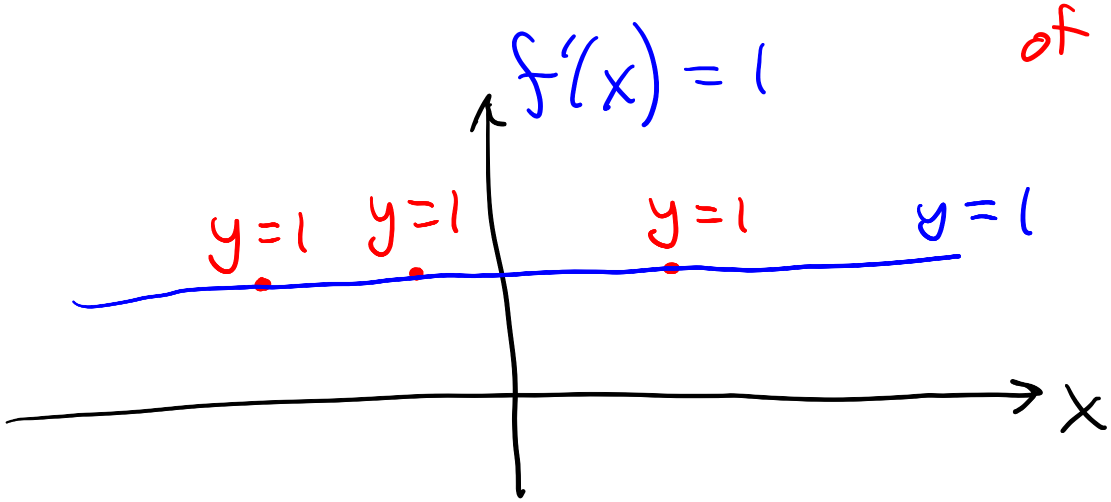
(B) Using a graph, discuss why the result makes sense.



All tangent lines on the graph of $f(x)$ have slope $m=1$

$\downarrow$

So all points on graph of $f'(x)$ will have $y=1$



[Example 8] (A) If $y = 1$, find y'

Solution: rewrite the function as a power function

$$y = 1 = x^0$$

Use the power rule to find $f'(x)$

$$\frac{d}{dx} x^0 \xleftarrow{} n=0 \quad = 0 \cdot x^{0-1} = 0 \cdot x^{-1} = 0$$

(B) Compare the result from **[Example 8]** to the result from ~~**[Example 1]**~~.

using constant function rule

$$y = 1 = \text{constant}$$

using the constant function rule

$$\frac{dy}{dx} = 0$$

End of Example

End of video