Subject for this video: Rewrite in Power Function Form, then Differentiate

Reading:

- **General:** Section 2.5, Basic Differentiation Properties
- **More specifically:** middle of p. 148 – top of p. 151, Parts of Examples 4,5,6

Homework:

Rewrite in Power Function Form, then Differentiate (2.5 #45,51,53,55,81)

Recall the Derivative Rules that we learned about in the previous videos.

The Constant Function Rule

This rule is used for finding the derivative of a *constant* function.

Two equation form: If $f(x) = c$ then $f'(x) = 0$.

Single equation form: $\frac{d}{dx} c = 0$

The Power Rule

This rule is used for finding the derivative of a *power* function.

Two equation form: If $f(x) = x^n$ then $f'(x) = nx^{n-1}$.

Single equation form: $\frac{d}{dx} x^n = nx^{n-1}$

The Sum and Constant Multiple Rule

If $f(x)$ and $g(x)$ are functions and a, b are constants, then

$$\frac{d}{dx}\big(af(x) + bg(x)\big) = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$$

Using prime notation, we could write

$$\big(af(x) + bg(x)\big)' = af'(x) + bg'(x)$$

In the previous video, we used the Sum and Constant Multiple Rule to find three fairly basic derivatives. In this video, we will do harder examples.

[Example 1] (similar to 2.5 # 45) Find $f'(x)$ for the function $f(x) = 7 + \dfrac{13}{x}$

Solution

The function f(x) is in positive exponent form.

Convert it to power function form

$$f(x) = 7 + \frac{13}{x} = 7 + 13 \cdot x^{-1}$$

Now take the derivative

$$f'(x) = \frac{d}{dx}\left(7 + 13x^{-1}\right)$$

use sum and constant multiple rule

$$= \underbrace{\frac{d}{dx}7}_{\text{constant function rule}} + \underbrace{13\frac{d}{dx}x^{-1}}_{\text{power rule with } n=-1} \leftarrow n=-1$$

$$= 0 + 13\left(-1 \cdot x^{-1-1}\right)$$

$$= -13x^{-2} \leftarrow \text{power function form}$$

$$= \frac{-13}{x^2} \leftarrow \text{converted to positive exponent form}$$

Remark: You have now seen to ways to find $f'(x)$ for the function $f(x) = 7 + \frac{13}{x}$

- In [Example 1] that we just finished, we used the *Sum and Constant Multiple Rule* and the *Power Rule*.
- In [Video for Homework H27 Example 1] (similar to exercise 2.4#35) we used the *Definition of the Derivative*, which was much harder.

The two methods gave the same result.

[Example 2] (similar to 2.5 # 53,55) Find $f'(x)$ for the function $f(x) = 7 + 13\sqrt{x}$

Solution Rewrite f(x) in power function form

$$f(x) = 7 + 13\sqrt{x} = 7 + 13 \cdot x^{1/2}$$

Now find the derivative

$$f'(x) = \frac{d}{dx}\left(7 + 13 \cdot x^{1/2}\right)$$

Sum and constant multiple rule

$$= \underbrace{\frac{d}{dx} 7}_{\text{constant function rule}} + \underbrace{13 \frac{d}{dx} x^{1/2}}_{\text{use power rule with } n = \frac{1}{2}}$$

$\leftarrow n = 1/2$

$$= 0 + 13\left(\frac{1}{2} \cdot x^{\frac{1}{2} - 1}\right)$$

$$= \frac{13x^{-1/2}}{2} = \frac{13}{2x^{1/2}}$$

Converted to positive exponent form

$$= \frac{13}{2\sqrt{x}}$$

Convert to radical form.

Remark: You have now seen to ways to find $f'(x)$ for the function $f(x) = 7 + 13\sqrt{x}$

- In **[Example 2]** that we just finished, we used the *Sum and Constant Multiple Rule* and the *Power Rule*.
- In **[Video for Homework H27 Example 2]** (similar to exercise 2.4#37) we used the *Definition of the Derivative*, which was much harder.

The two methods gave the same result.

[Example 3] (similar to 2.5#51,53,55) Find $h'(t)$ if $h(t) = \frac{7}{5\sqrt[3]{t}} + \frac{3}{11t^{2/5}}$

Solution Start by rewriting $h(t)$ in power function form

$$h(t) = \frac{7}{5\sqrt[3]{x}} + \frac{3}{11t^{2/5}} = \frac{7}{5}\cdot\frac{1}{\sqrt[3]{x}} + \frac{3}{11}\cdot\frac{1}{t^{2/5}} = \frac{7}{5}\cdot x^{-1/3} + \frac{3}{11}\cdot t^{-2/5}$$

Separate the constants

Convert to power function form

Now find the derivative

$$h'(t) = \frac{d}{dt}\left(\frac{7}{5}\cdot t^{-1/3} + \frac{3}{11}t^{-2/5}\right)$$

Apply the Sum and Constant Multiple Rule

$$= \frac{7}{5}\cdot\frac{d}{dt}t^{-1/3} + \frac{3}{11}\cdot\frac{d}{dt}t^{-2/5}$$

$n = -\frac{1}{3}$ $n = -\frac{2}{5}$

use power rule

$$= \frac{7}{5}\left(-\frac{1}{3}\cdot t^{-\frac{1}{3}-1}\right) + \frac{3}{11}\cdot\left(\frac{-2}{5}\cdot t^{-\frac{2}{5}-1}\right)$$

$$= \frac{-7}{15}\cdot t^{-4/3} + \left(\frac{-6}{55}\right)\cdot t^{-7/5}$$

Convert to positive exponent form

$$= -\frac{7}{15t^{4/3}} - \frac{6}{55t^{7/5}}$$

[Example 4] (similar to to 2.5#81): Find y' if $y = \dfrac{2x^5 - 4x^3 + 2x}{x^3}$

Solution Rewrite the function in power function form.

$$y = \frac{2x^5 - 4x^3 + 2x}{x^3} = \frac{2x^5}{x^3} - \frac{4x^3}{x^3} + \frac{2x}{x^3} = 2x^2 - 4 + \frac{2}{x^2}$$

$$= \underline{2x^2 - 4 + 2x^{-2}}$$

this is in power function form

Now find the derivative.

$$y' = \frac{d}{dx}\left(2x^2 - 4 + 2x^{-2}\right)$$

Start by using the Sum and Constant Multiple Rule

$$= 2\frac{d}{dx}x^2 - \frac{d}{dx}4 + 2\frac{d}{dx}x^{-2}$$

Constant Function Rule

$$= 2(2x^{2-1}) - 0 + 2(-2x^{-2-1})$$

$$= 4x^1 - 4x^{-3}$$

$$= 4x - \frac{4}{x^3}$$

Simplify, and convert to positive exponent form.

End of Example

End of Video