Subject for this video: Tangent Line and Applied Problems involving Exponential Functions

Reading:

- **General:** Section 3.2 Derivatives of Exponential and Logarithmic Functinos
- **More Specifically:** Page 192 - 193, Examples 4, 5

Homework:

H41: Tangent Line and Applied Problems involving Exponential Functions (3.2#33,67,75)

Recall the Derivative Rules that we learned about in previous videos.

The Constant Function Rule:	$\frac{d}{dx}c = 0$
The Power Rule:	$\frac{d}{dx}x^n = nx^{n-1}$
The Sum and Constant Multiple Rule:	$\frac{d}{dx}\big(af(x) + bg(x)\big) = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$
Exponential Function Rule #1:	$\frac{d}{dx}e^{(x)} = e^{(x)}$
Exponential Function Rule #2:	$\frac{d}{dx}e^{(kx)} = ke^{(kx)}$
Exponential Function Rule #3:	$\frac{d}{dx}b^{(x)} = b^{(x)} \cdot \ln(b)$

[Example 1] (Similar to 3.2#33) For the function: $f(x) = 2e^{(x)} - 3x$.

(A) Find equation of line tangent to graph of f at $x = 0$.

Solution We need to build the equation $\underbrace{(y - f(a)) = f'(a)\cdot(x - a)}$

point slope form of the equation for the tangent line.

Get parts

$a = 0$ the x coordinate of the point of tangency

$f(a) = f(0) \underset{\uparrow}{=} 2e^{(0)} - 3(0) = 2\cdot 1 - 0 = 2$ the y coordinate of the point of tangency

sub $x = 0$ into $f(x)$

$f'(x) = \frac{d}{dx}\left(2e^{(x)} - 3x\right) \underset{\uparrow}{=} 2\frac{d}{dx}e^{(x)} - 3\frac{d}{dx}x \xrightarrow{\text{rule \#1}} = 2\cdot\left(e^{(x)}\right) - 3\left(1\cdot x^{1-1}\right) = 2e^{(x)} - 3$

Sum and constant multiple rule

power rule with $n = 1$

$f'(a) = f'(0) \underset{\uparrow}{=} 2e^{(0)} - 3 = 2\cdot 1 - 3 = -1$ the slope m of the tangent line

sub $x=0$ into $f'(x)$

Substitute parts into tangent line equation

$$y - f(a) = f'(a)(x - a)$$

$$(y - 2) = (-1)(x - 0)$$

Equation for the tangent line in point slope form

Convert to slope intercept form. (Solve for y)

$$y - 2 = -x$$

$$y = -x + 2$$

equation for the tangent line in slope intercept form

(B) Find equation of line tangent to graph of f at $x = 1$.

Solution we need to build $(y - f(a)) = f'(a)(x - a)$

Get Parts

$a = 1$ the x coordinate of the point of tangency

$f(a) = f(1) = 2e^{(1)} - 3(1) = 2e - 3$ the y coordinate of the point of tangency

sub $x=1$ into
$f(x) = 2e^{(x)} - 3x$

$f'(a) = f'(1) = 2e^{(1)} - 3 = 2e - 3$ the slope of the tangent line

Sub $x=1$ into
$f'(x) = 2e^{(x)} - 3$

Substitute parts into the tangent line equation

$$(y - (2e-3)) = (2e-3)(x-1)$$

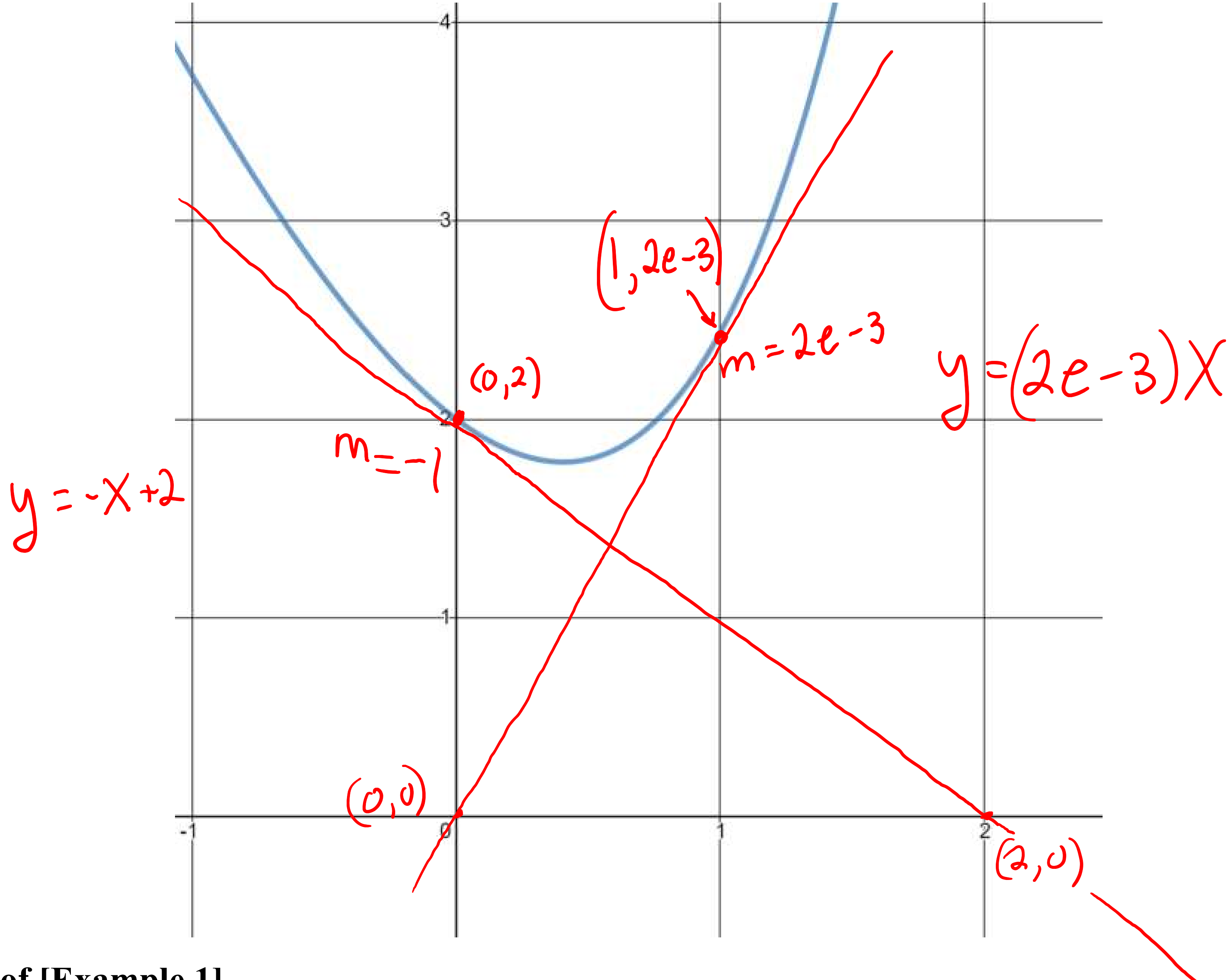
convert to slope intercept form by solving for x

$$y - \cancel{(2e-3)} = (2e-3)x - \cancel{(2e-3)}$$

$$y = (2e-3)x$$

Slope intercept form of the equation for the tangent line

(C) Illustrate the results of **(A)**,**(B)** on the given graph of $f(x) = 2e^{(x)} - 3x$.



End of [Example 1].

[Example 2] P dollars is deposited into an account that has continuously compounded interest with interest rate r.

(A) What is the value at t years?

Solution: $A = Pe^{(rt)}$

(B) What is the instantaneous rate of change of the balance at time t years?

Solution: we need to find A'

function form $A(t) = Pe^{(rt)}$

So $A'(t) = \frac{d}{dt} Pe^{(rt)} \underset{\text{constant multiple rule}}{=} P \cdot \frac{d}{dt} e^{(rt)} \underset{\text{rule \#2 with } k=r}{=} P\left(r \cdot e^{(rt)}\right) \underset{\text{clean up}}{=} r \cdot Pe^{(rt)}$

$$A'(t) = r \cdot P \cdot e^{(rt)} = r \cdot A(t)$$

End of [Example 2]

[Example 3] (Similar to 3.2#75)

An investment of $1000 earns interest at a nominal rate of 7% compounded continuously.

(A) What is the value at 10 years?

(Give an exact answer in symbols and a decimal approximation. Include the correct units.)

Solution $A(t) = Pe^{(rt)}$

$P = 1000$ the principal

$r = .07$

$t = 10$

$A =$ unknown, find A

$$A = 1000\,e^{(.07\cdot 10)} = \underbrace{1000\,e^{(0.7)}}_{\text{exact answer}} \approx \$2013^{\underline{75}}$$

use wolfram alpha

(B) How fast is the value growing at time $t = 10$ years?

(Give an exact answer in symbols and a decimal approximation. Include the correct units.)

Solution We need to find $A'(10)$

$$A'(10) = (.07)1000e^{(.07 \cdot 10)} = \underbrace{70e^{(.7)}}_{\text{exact answer}} \approx 140.97 \text{ dollars per year}$$

sub $t=10$ into $A'(t) = rPe^{(rt)}$

use Wolfram alpha

(C) How fast is the value growing when the value is $8000?

(Give an exact answer. Include the correct units.)

Observe: We are being asked to find the value of $A'(t)$, but we are not given the value of t.

hard

One might think that we would have to do this:

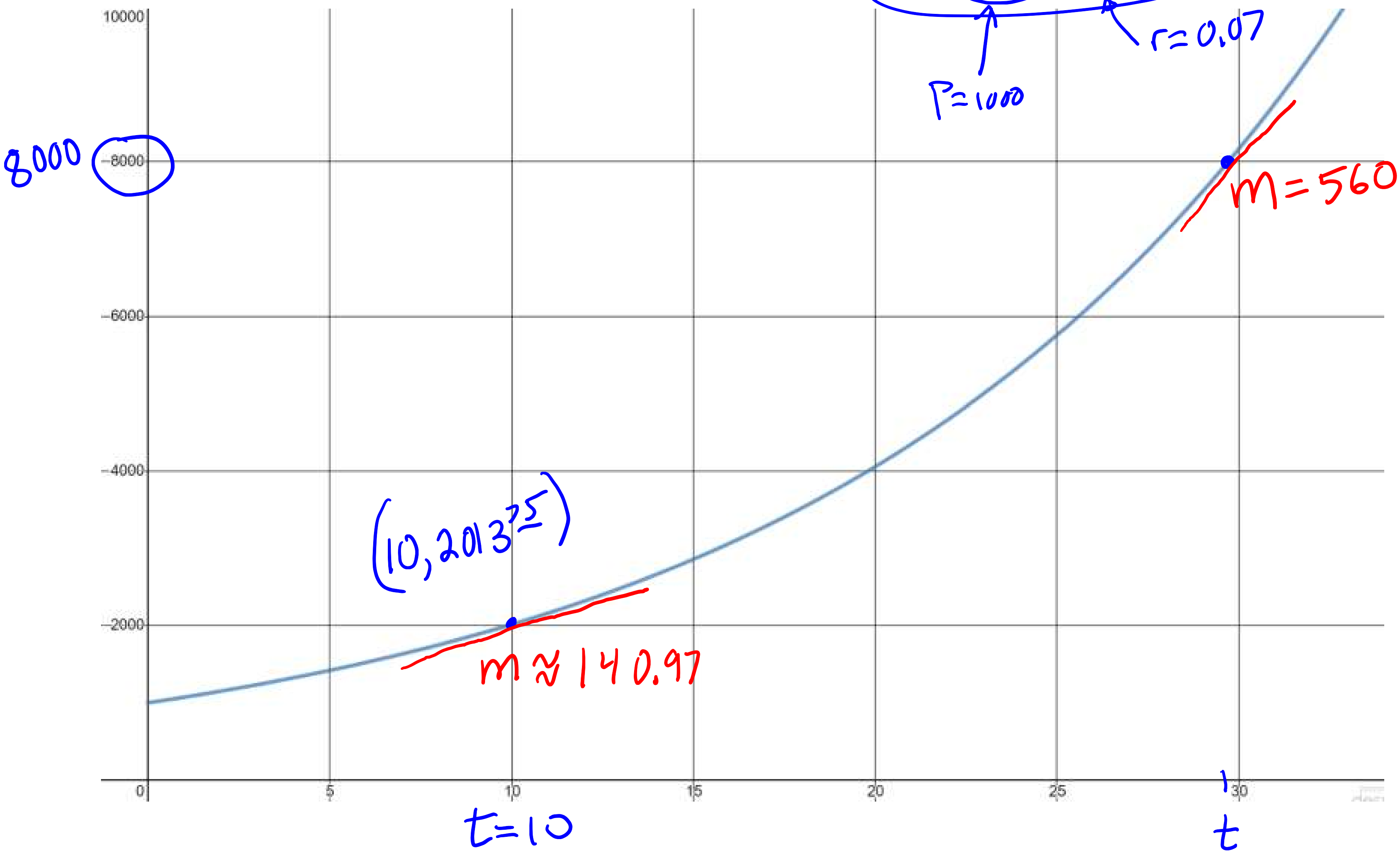
Use fact that $A = 8000$ to find t (hard)

Use that value of t to find $A'(t)$ (hard)

The clever solution is to recall what we know about $A'(t)$

$$A'(t) = r \cdot A(t) = .07(8000) = 560 \text{ dollars per yer}$$

(D) Illustrate the results of **(A),(B),(C)** on the given graph of $A = 1000e^{(0.07 \cdot t)}$



End of [Example 3]

[Example 4] (Similar to 3.2#67)

The estimated salvage value of a new luxury car is described by the function

$$S(t) = 65{,}000(0.85)^{(t)}$$

In this function,

t is the time in years since the car was purchased.

$S(t)$ is the salvage value at time t. (in dollars)

(A) What was the purchase price of the car?

$$S(0) = 65{,}000(0.85)^{(0)} = 65{,}000(1) = \$65{,}000$$

(B) What is the value of the car at time $t = 6$ years?

(Give an exact answer in symbols and a decimal approximation. Include the correct units.)

$$S(6) = \underbrace{65{,}000(0.85)^{(6)}}_{\text{exact answer}} \approx \$24515$$

Sub $t=6$ into $S(t)$

Wolfram Alpha

(C) What is the rate of change of the value of the car at time $t = 6$ years?

(Give an exact answer in symbols and a decimal approximation. Include the correct correct units.)

We need to find $S'(6)$

Strategy: • find $S'(t)$

• Substitute $t=6$ to get $S'(6)$

$$S'(t) = \frac{d}{dt}\,65{,}000(0.85)^t = 65{,}000\left(\frac{d}{dt}0.85^{(t)}\right) =$$

constant multiple rule

$$= 65{,}000\left(0.85^{(t)} \cdot \ln(0.85)\right) = (65{,}000)\ln(0.85)\cdot(0.85)^{(t)}$$

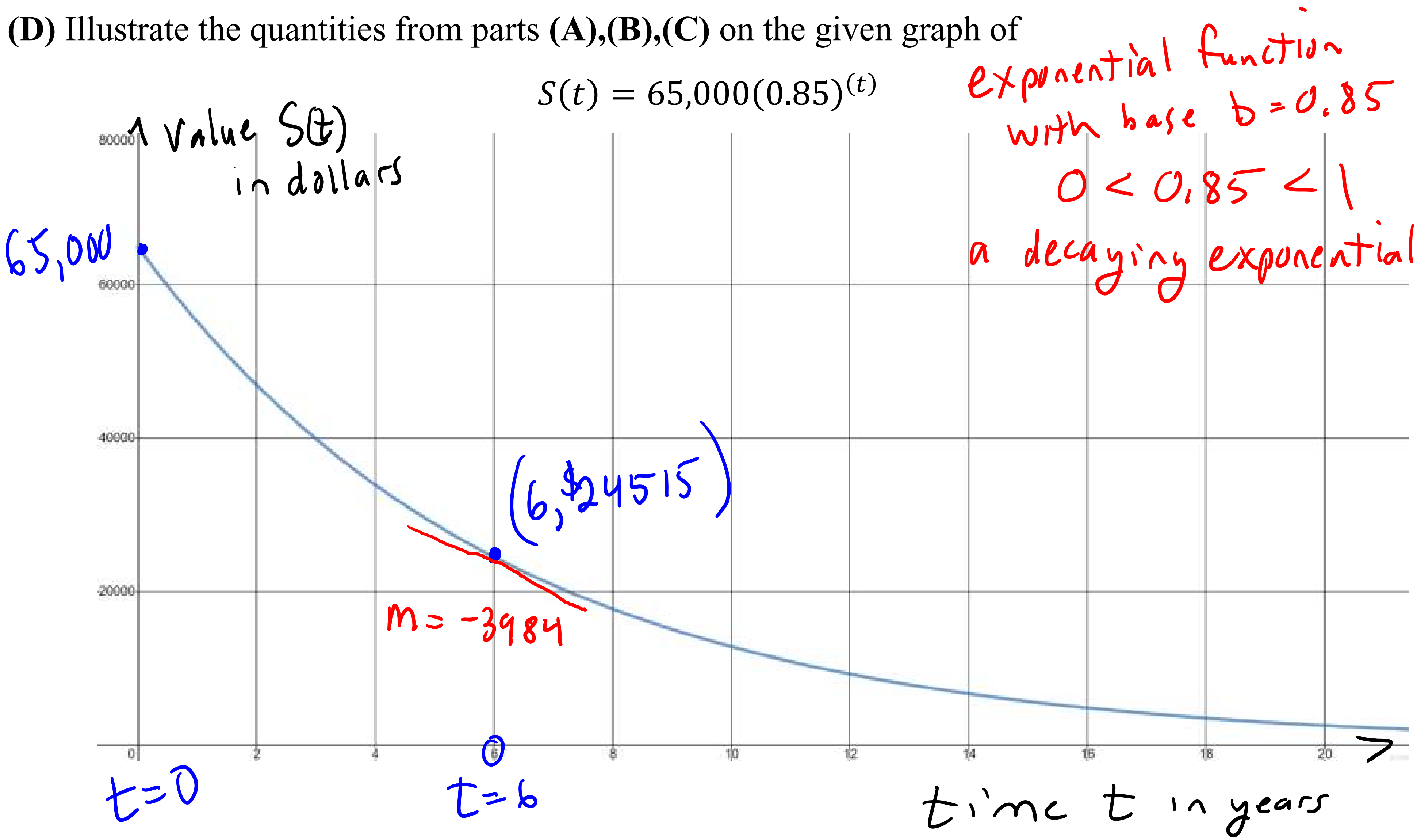
use Rule #3 with $b = 0.85$

$$S'(6) = \underbrace{65{,}000\ln(0.85)\cdot(0.85)^{(6)}}_{\text{exact answer}} \approx -3984 \text{ dollars per year}$$

use wolfram alpha

(D) Illustrate the quantities from parts **(A)**,**(B)**,**(C)** on the given graph of

$$S(t) = 65{,}000(0.85)^{(t)}$$



End of [Example 4]

End of Video