Subject for this video: Tangent Line Problems Involving Logarithmic Functions

Reading:

- **General:** Section 3.2 Derivatives of Exponential and Logarithmic Functions
- **More Specifically:** The book does not discuss tangent lines in Section 3.2, and there are no similar examples.

Homework:

H44: Derivatives of Logarithmic Functions (3.2#31,35)

Recall the Derivative Rules that we learned about in previous videos.

The Constant Function Rule:	$\frac{d}{dx}c = 0$
The Power Rule:	$\frac{d}{dx}x^n = nx^{n-1}$
The Sum and Constant Multiple Rule:	$\frac{d}{dx}\big(af(x) + bg(x)\big) = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$
Exponential Function Rule #1:	$\frac{d}{dx}e^{(x)} = e^{(x)}$
Exponential Function Rule #2:	$\frac{d}{dx}e^{(kx)} = ke^{(kx)}$
Exponential Function Rule #3:	$\frac{d}{dx}b^{(x)} = b^{(x)} \cdot \ln(b)$
Logarithmic Function Rule #1	$\frac{d}{dx}\ln(x) = \frac{1}{x}$
Logarithmic Function Rule #2	$\frac{d}{dx}\log_b(x) = \frac{1}{x\ln(b)}$

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[Example]

(A) Find equation of line tangent to $f(x) = 5 + \ln(x^3)$ at $x = e^2$.

We need to find $(y - f(a)) = f'(a) \cdot (x - a)$ — Point slope form of the equation of the tangent line

Get Parts

$a = e^2$ — the x coordinate of the point of tangency

$f(a) = f(e^2) = 5 + \ln((e^2)^3) = 5 + \ln(e^6) = 5 + 6\ln(e) = 5 + 6 \cdot 1 = 11$ — y coord of point of tangency

Sub $x = e^2$ into $f(x)$; simplify $(e^a)^b = e^{a \cdot b}$; property $\ln(a^b) = b\ln(a)$; $\ln(e) = 1$

Rewrite $f(x) = 5 + \ln(x^3) = 5 + 3\ln(x)$ — use property $\ln(a^b) = b\ln(a)$

$f'(x) = \frac{d}{dx} 5 + 3\ln(x) = \frac{d}{dx}5 + 3\frac{d}{dx}\ln(x) = 0 + 3\left(\frac{1}{x}\right) = \frac{3}{x}$

Sum & constant multiple rule

$f'(a) = \frac{3}{e^2}$ — Slope of the tangent line

Sub $a = e^2$ into $f'(x)$

Substitute parts into the tangent line equation

$$(y - f(a)) = f'(a)(x - a)$$

$$(y - 11) = \left(\frac{3}{e^2}\right)(x - e^2)$$

equation for the tangent line in point slope form

Convert to slope intercept form. (Solve for y)

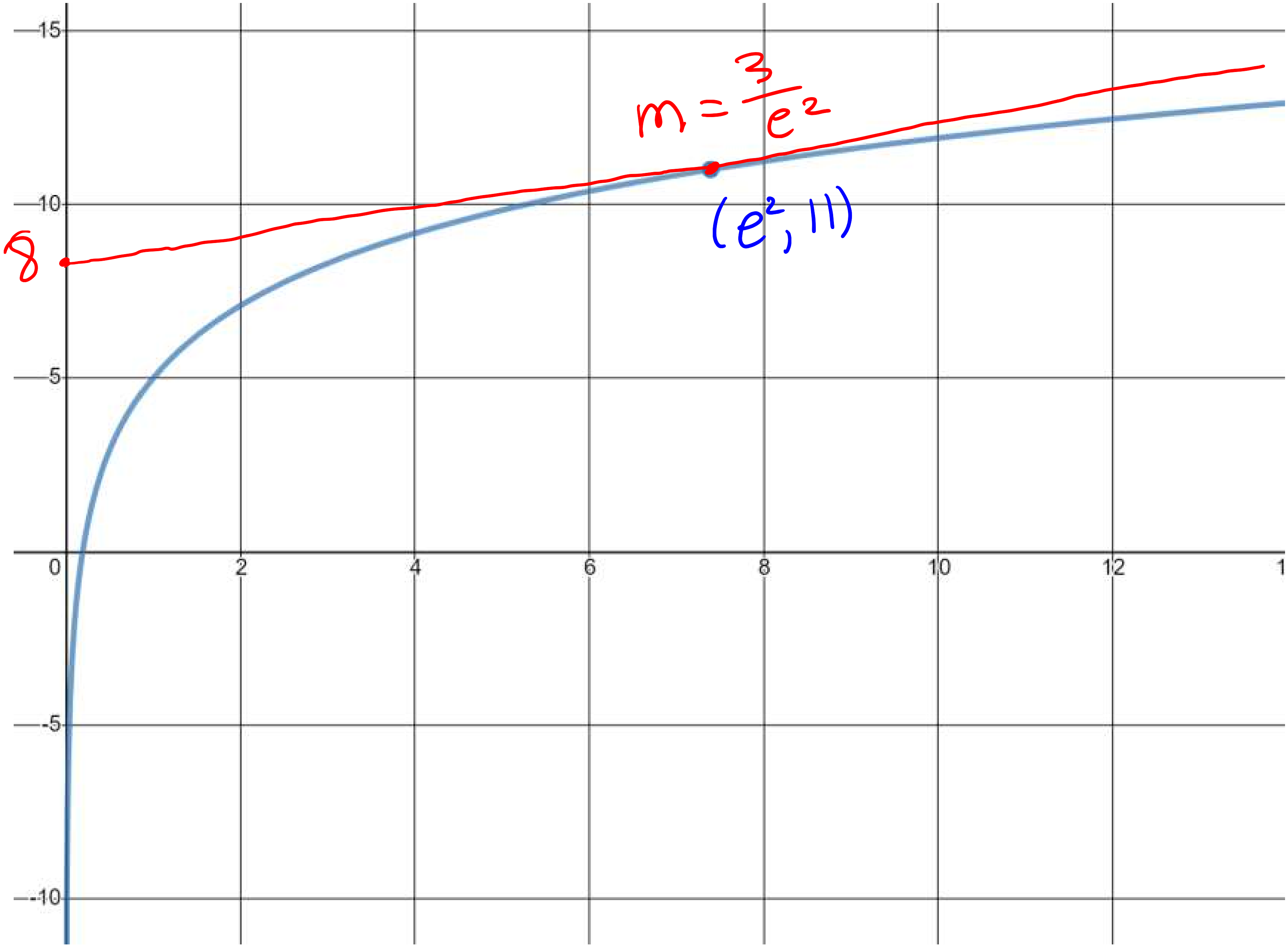
$$y - 11 = \left(\frac{3}{e^2}\right)(x - e^2) = \left(\frac{3}{e^2}\right)\cdot x - \left(\frac{3}{\cancel{e^2}}\right)\cancel{e^2}$$

$$= \left(\frac{3}{e^2}\right)\cdot x - 3$$

$$y = \left(\frac{3}{e^2}\right)\cdot x + 8$$

equation for the tangent line in slope intercept form.

(B) Illustrate your solution on the given graph of $f(x) = 5 + \ln(x^3)$



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