Subject for this video: Derivatives of Products

Reading:

- **General:** Section 3.3 Derivatives of Products and Quotients
- **More Specifically:** Pages 196 – 198, Examples 1,2,3

Homework:

H45: Differentiating Products (3.3#17,19,21,55)

Recall the Derivative Rules that we learned about in previous videos.

The Constant Function Rule:	$\frac{d}{dx}c = 0$
The Power Rule:	$\frac{d}{dx}x^n = nx^{n-1}$
The Sum and Constant Multiple Rule:	$\frac{d}{dx}\big(af(x) + bg(x)\big) = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$
Exponential Function Rule #1:	$\frac{d}{dx}e^{(x)} = e^{(x)}$
Exponential Function Rule #2:	$\frac{d}{dx}e^{(kx)} = ke^{(kx)}$
Exponential Function Rule #3:	$\frac{d}{dx}b^{(x)} = b^{(x)} \cdot \ln(b)$
Logarithmic Function Rule #1	$\frac{d}{dx}\ln(x) = \frac{1}{x}$
Logarithmic Function Rule #2	$\frac{d}{dx}\log_b(x) = \frac{1}{x\ln(b)}$

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New Rule: The Product Rule

Consider a product of functions, $f(x) = g(x) \cdot h(x)$.

Question: How is $f'(x)$ related to $g'(x)$ and $h'(x)$?

The good news is that there is an obvious relationship.

The bad news is that the obvious relationship is wrong.

The obvious relationship: ~~If $f(x) = g(x) \cdot h(x)$ then $f'(x) = g'(x) \cdot h'(x)$~~ wrong!

How do we know it is wrong? Consider this easy example.

[Example 1] Let $f(x) = 5x^2$

If we find $f'(x)$ using our established derivative rules, we find

power function with $n=1$

$$f'(x) = \frac{d}{dx} 5x^2 \underset{\uparrow}{=} 5 \frac{d}{dx} x^2 \underset{\uparrow}{=} 5(2x^{2-1}) = 10x$$

constant multiple rule   power rule with $n=1$

But if we compute $f'(x)$ using the obvious (bad) method, we find

$$\cancel{f'(x) = \frac{d}{dx} 5x^2 = \left(\frac{d}{dx} 5\right) \cdot \left(\frac{d}{dx} x^2\right) = (0)(2x) = 0}$$ clearly wrong!

The correct relationship between $f'(x)$ and $g'(x)$ and $h'(x)$ is given by the Product Rule.

The Product Rule

This rule is used for finding the derivative of a *product of functions*.

Two equation form: If $f(x) = g(x) \cdot h(x)$ then $f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$

Single equation form: $\frac{d}{dx} g(x) \cdot h(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$

[Example 2] (Similar to 3.3#21) $f(x) = (-3x^2 + 13x - 5)(3x - 2)$.

Find $f'(x)$ using the product rule. Simplify your answer.

$$f'(x) = \frac{d}{dx}\left((-3x^2+13x-5)(3x-2)\right)$$

Product rule

$$= \left(\frac{d}{dx}(-3x^2+13x-5)\right)\cdot(3x-2) + (-3x^2+13x-5)\left(\frac{d}{dx}(3x-2)\right)$$

$$= \left(-3(2x)+13(1)-0\right)\cdot(3x-2) + (-3x^2+13x-5)(3(1)-0)$$

(used the power rule and the constant function rule)

$$= (-6x+13)(3x-2) + (-3x^2+13x-5)(3)$$

$$= -18x^2+39x+12x-26 -9x^2+39x-15$$

$$= -27x^2+90x-41$$

[Example 3] (Similar to 3.3#55) Let $f(x) = (-3x^2 + 13x - 5)e^{(x)}$

(A) Find $f'(x)$ and simplify.

$$f'(x) = \frac{d}{dx}\left((-3x^2+13x-5)e^{(x)}\right)$$

product rule

$$= \left(\frac{d}{dx}(-3x^2+13x-5)\right)\cdot e^{(x)} + (-3x^2+13x-5)\frac{d}{dx}e^{(x)}$$

$$= (-6x+13)\cdot \underbrace{e^{(x)}} + (-3x^2+13x-5)\left(\underbrace{e^{(x)}}\right)$$

$$= \left[(-6x+13) + (-3x^2+13x-5)\right]\underbrace{e^{(x)}}$$

$$= \left[-3x^2+7x+8\right]e^{(x)}$$

(B) Find $f'(0)$ and simplify.

$$f'(x) = \left[-3x^2 + 7x + 8\right] e^{(x)}$$

$$f'(\ \ ) = \left[-3(\ \ )^2 + 7(\ \ ) + 8\right] e^{(\ \ )}$$

$$f'(0) = \left[-3(0)^2 + 7(0) + 8\right] e^{(0)} = [8] e^{(0)} = 8 \cdot 1 = 8$$

substitute x=0 into f'(x)

(C) Find $f'(1)$ and simplify.

$$f'(1) = \left[-3(1)^2 + 7(1) + 8\right] e^{(1)} = -3(1) + 7 + 8] \cdot e$$

sub x=1 into f'(x)

$$= [12] e$$

$$= 12e$$

[Example 4] (Similar to 3.3#19) Let $f(x) = 5x^7 \ln(x)$

(A) Find $f'(x)$ and simplify.

note: don't do this $f'(x) = \left(\frac{d}{dx} 5x^7\right) \cdot \left(\frac{d}{dx} \ln(x)\right)$!!

use product rule

$$f'(x) = \frac{d}{dx}\left(5x^7 \cdot \ln(x)\right) = \left(\frac{d}{dx} 5x^7\right) \cdot \ln(x) + 5x^7 \cdot \left(\frac{d}{dx} \ln(x)\right)$$

Product rule

$$= \left(5 \cdot (7x^{7-1})\right) \ln(x) + 5x^7 \cdot \left(\frac{1}{x}\right)$$

$$= 35x^6 \ln(x) + 5x^6$$

$$= \underbrace{5x^6}\cdot 7\ln(x) + \underbrace{5x^6}\cdot 1$$

$$= 5x^6 \left[7\ln(x) + 1\right]$$

(B) Find $f'(1)$ and simplify.

$$f'(x) = 5x^6[7\ln(x) + 1]$$

$$f'(1) = 5(1)^6[7\ln(1) + 1] = 5\cdot 1[7\cdot 0 + 1] = 5[1] = 5$$

sub $x=1$ into $f'(x)$

$\ln(1) = 0$

(because $e^{(0)} = 1$)

(C) Find $f'(e)$ and simplify.

$$f'(e) = 5(e)^6[7\ln(e) + 1] = 5e^6[7\cdot 1 + 1] = 5e^6[8]$$

sub $x = e$ into $f'(x)$

$\ln(e) = 1$

(because $e^{(1)} = e$)

$$= 40e^6$$