Subject for this video: A trick that works for derivatives of some quotients:

Rewrite First to Eliminate the Quotient

Reading:

- **General:** Section 3.3 Derivatives of Products and Quotients
- **More Specifically:** Page 199, Example 4C~~5~~

Homework:

H47: Trick: Rewrite First to Eliminate the Quotient (3.3#59*,73)

Recall the Derivative Rules that we learned about in previous videos.

The Constant Function Rule:	$\frac{d}{dx}c = 0$
The Power Rule:	$\frac{d}{dx}x^n = nx^{n-1}$
The Sum and Constant Multiple Rule:	$\frac{d}{dx}\big(af(x) + bg(x)\big) = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$
Exponential Function Rule #1:	$\frac{d}{dx}e^{(x)} = e^{(x)}$
Exponential Function Rule #2:	$\frac{d}{dx}e^{(kx)} = ke^{(kx)}$
Exponential Function Rule #3:	$\frac{d}{dx}b^{(x)} = b^{(x)} \cdot \ln(b)$
Logarithmic Function Rule #1:	$\frac{d}{dx}\ln(x) = \frac{1}{x}$
Logarithmic Function Rule #2:	$\frac{d}{dx}\log_b(x) = \frac{1}{x\ln(b)}$
The Product Rule:	$\frac{d}{dx}g(x) \cdot h(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$

.

And the *Quotient Rule* that we learned about in the previous video

The Quotient Rule

This rule is used for finding the derivative of a *quotient of functions*.

Two equation form:

If

$$f(x) = \frac{top(x)}{bottom(x)}$$

then

$$f'(x) = \frac{top'(x)bottom(x) - top(x)bottom'(x)}{\left(bottom(x)\right)^2}$$

Single equation form:

$$\frac{d}{dx}\left(\frac{top(x)}{bottom(x)}\right) = \frac{top'(x)bottom(x) - top(x)bottom'(x)}{\left(bottom(x)\right)^2}$$

[Example 1] (similar to 3.3#73)

$$Let\ \ f(x) = \frac{x^7 + 13}{x^7}$$

The goal is to find $f'(x)$ by two methods.

(A) Find $f'(x)$ by using the quotient rule.

$$f'(x) = \frac{\left(\frac{d}{dx} x^7 + 13\right) \cdot x^7 - (x^7 + 13) \frac{d}{dx} x^7}{(x^7)^2}$$

quotient rule

$$= \frac{(7x^6 + 0) \cdot x^7 - (x^7 + 13) \cdot (7x^6)}{(x^7)^2}$$

$$= \frac{\cancel{7x^{13}} - (\cancel{7x^{13}} + 91x^6)}{x^{14}}$$

using $(a^b)^c = a^{b \cdot c}$

$$= -\frac{91x^6}{x^{14}}$$

$$= -\frac{91}{x^8}$$

(B) Start over. First simplify $f(x)$. Then find $f'(x)$ using easier derivative rules.

$$f(x) = \frac{x^7+13}{x^7} = \frac{x^7}{x^7} + \frac{13}{x^7} = 1 + 13x^{-7}$$

Convert from positive exponent form to power function form

Now find the derivative

$$f'(x) = \frac{d}{dx}\left(1 + 13x^{-7}\right) = \frac{d}{dx}1 + 13\frac{d}{dx}x^{-7}$$

sum and constant multiple rule

power function with $n=-7$

use power rule with $n=-7$

$$= 0 + 13\cdot\left(-7\cdot x^{-7-1}\right)$$

$$= -91x^{-8}$$

Convert from power function form to positive exponent form

$$= -91\cdot\frac{1}{x^8}$$

$$= \frac{-91}{x^8}$$

Remark: Solution B, where we simplified $f(x)$ first, is much easier than solution A, which needed the quotient rule.