Subject for this video: Tangent Line and Applied Problems Involving Quotients

Reading:

- **General:** Section 3.3 Derivatives of Products and Quotients
- **More Specifically:**
  - There is no discussion of tangent line problems involving quotients in Section 3.3. There is an example involving a product (Example 2). But a specific book example involving quotients is not really needed: We have seen tangent line examples in book sections 2.4, 2.5, and 3.2. All the tangent line problems are solved the same way. Only the particular technique used to find the derivative varies.
  - There is an applied problem involving a quotient on page 201: Example 6

Homework:

H48: Tangent Line Problems Involving Quotients (3.3#63,93)

Recall the Derivative Rules that we learned about in previous videos.

The Constant Function Rule:	$\frac{d}{dx}c = 0$
The Power Rule:	$\frac{d}{dx}x^n = nx^{n-1}$
The Sum and Constant Multiple Rule:	$\frac{d}{dx}\big(af(x) + bg(x)\big) = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$
Exponential Function Rule #1:	$\frac{d}{dx}e^{(x)} = e^{(x)}$
Exponential Function Rule #2:	$\frac{d}{dx}e^{(kx)} = ke^{(kx)}$
Exponential Function Rule #3:	$\frac{d}{dx}b^{(x)} = b^{(x)} \cdot \ln(b)$
Logarithmic Function Rule #1:	$\frac{d}{dx}\ln(x) = \frac{1}{x}$
Logarithmic Function Rule #2:	$\frac{d}{dx}\log_b(x) = \frac{1}{x\ln(b)}$
The Product Rule:	$\frac{d}{dx}g(x) \cdot h(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$

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And the *Quotient Rule* that we learned about recently

The Quotient Rule

This rule is used for finding the derivative of a *quotient of functions*.

Two equation form:

If

$$f(x) = \frac{top(x)}{bottom(x)}$$

then

$$f'(x) = \frac{top'(x)bottom(x) - top(x)bottom'(x)}{\left(bottom(x)\right)^2}$$

Single equation form:

$$\frac{d}{dx}\left(\frac{top(x)}{bottom(x)}\right) = \frac{top'(x)bottom(x) - top(x)bottom'(x)}{\left(bottom(x)\right)^2}$$

[Example 1] (similar to 3.3#63)

Let $f(x) = \dfrac{2x-5}{2x-3}$

(A) Find the equation of the line tangent to the graph of $f(x)$ at $x = 3$.

We need to build the equation $(y - f(a)) = f'(a)\cdot(x-a)$ point slope form of the equation for the tangent line

Get Parts

$a = 3$ the x coordinate of the point of tangency

$f(a) = f(3) = \dfrac{2(3)-5}{2(3)-3} = \dfrac{6-5}{6-3} = \dfrac{1}{3}$ the y coordinate of the point of tangency

$$f'(x) = \frac{d}{dx}\left(\frac{2x-5}{2x-3}\right) = \frac{\left(\frac{d}{dx}(2x-5)\right)\cdot(2x-3) - (2x-5)\frac{d}{dx}(2x-3)}{(2x-3)^2}$$

quotient rule

Remember: we cannot cancel $(2x-3)$.

$$= \frac{(2)\cdot(2x-3) - (2x-5)(2)}{(2x-3)^2}$$

$$= \frac{(\cancel{4x}-6) - (\cancel{4x}-10)}{(2x-3)^2}$$

$$= \frac{4}{(2x-3)^2}$$

$$f'(a) = f'(3) = \frac{4}{(2(3)-3)^2} = \frac{4}{(3)^2} = \frac{4}{9}$$ slope m of the tangent line.

Sub $x=3$ into $f'(x)$

Substitute parts into the equation

$$\left(y - \frac{1}{3}\right) = \frac{4}{9}(x - 3)$$ The point slope form of the tangent line equation

Convert to slope intercept form by solving for y.

$$y - \frac{1}{3} = \frac{4}{9}(x-3) = \left(\frac{4}{9}\right)x - \left(\frac{4}{9}\right)3$$

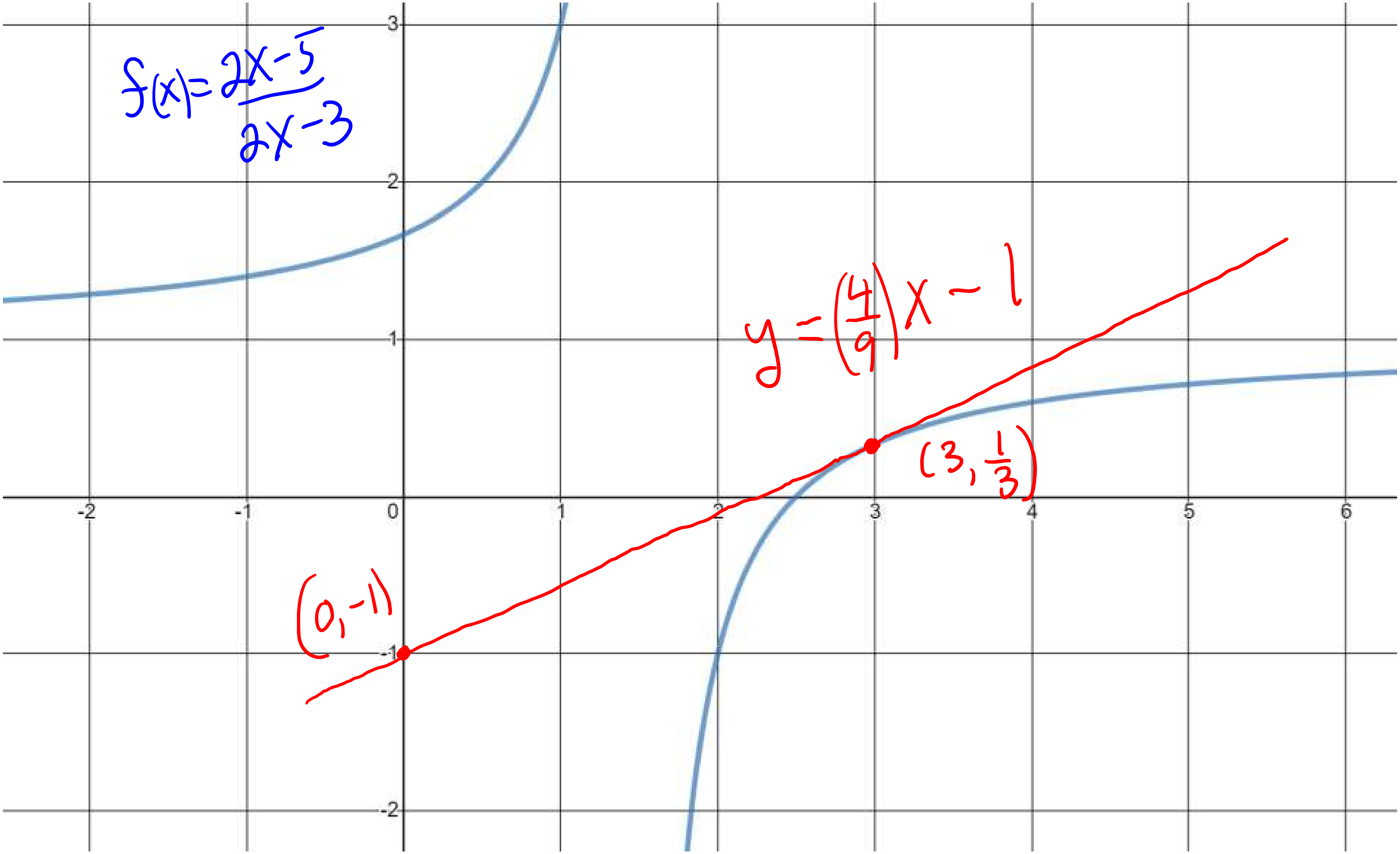
$$= \left(\frac{4}{9}\right)x - \frac{4}{3}$$

$$y = \left(\frac{4}{9}\right)x - \frac{4}{3} + \frac{1}{3}$$

$$= \left(\frac{4}{9}\right)x - \frac{3}{3}$$

$$\boxed{y = \left(\frac{4}{9}\right)x - 1}$$ Slope intercept form of the tangent line equation.

(B) Illustrate your result of **(A)** on the given graph of $f(x)$



[Example 2] (similar to 3.3#93) Sales of a game are described by the function

$$S(t) = \frac{50t}{t+4}$$

where t is the time (in months) since the game was introduced

and $S(t)$ is the total sales (in thousands of games) at time t.

(A) Find $S(6)$.

$$S(6) = \frac{50(6)}{(6)+4} = \frac{300}{10} = 30$$

sub $t=6$
into $S(t)$

(B) Find $S'(t)$. Show all details clearly, use correct notation, and simplify your answer.

$$S'(t) = \frac{d}{dt}\left(\frac{50t}{t+4}\right) = \frac{\left(\frac{d}{dt}(50t)\right)(t+4) - 50t\left(\frac{d}{dt}(t+4)\right)}{(t+4)^2}$$

quotient rule

cannot cancel the t+4!!

$$= \frac{(50)(t+4) - 50t(1)}{(t+4)^2}$$

$$= \frac{\cancel{50t} + 200 - \cancel{50t}}{(t+4)^2}$$

$$= \frac{200}{(t+4)^2}$$

(C) Find $S'(6)$.

$$S'(6) = \frac{200}{((6)+4)^2} = \frac{200}{(10)^2} = \frac{200}{100} = 2$$

sub $t=6$
into $S'(t)$

(D) Write a brief interpretation of the answers from **(A)** and **(C)**.

from (A) $S(6) = 30$

from (C) $S'(6) = 2$

At time $t = 6$ months since the game was introduced, a total of 30 thousand games have been sold.

and the total sales are increasing at a rate roughly 2 thousand games per month.

(that is, games are selling at a rate of 2 thousand games per month at that time)

(E) Use the results of **(D)** to estimate the total sales after 7 months.

total sales at $t=7$ $\approx$ total sales at $t=6$ + rate at which games are selling at $t=6$

$$S(7) \approx S(6) + S'(6)$$

Recall: exact change in sales: $\Delta S = S(7) - S(6)$

approximate change in sales $= S'(6)$

$$\Delta S \approx S'(6)$$

exact change $\approx$ approximate change

$$S(7) - S(6) \approx S'(6)$$

$$S(7) \approx S(6) + S'(6)$$

That is, total sales at $t=7$ months will be roughly 30 thousand + 2 thousand = 32 thousand games

(F) According to the model, roughly how many games will eventually sell?

That is, what is $\lim_{t \to \infty} S(t)$?

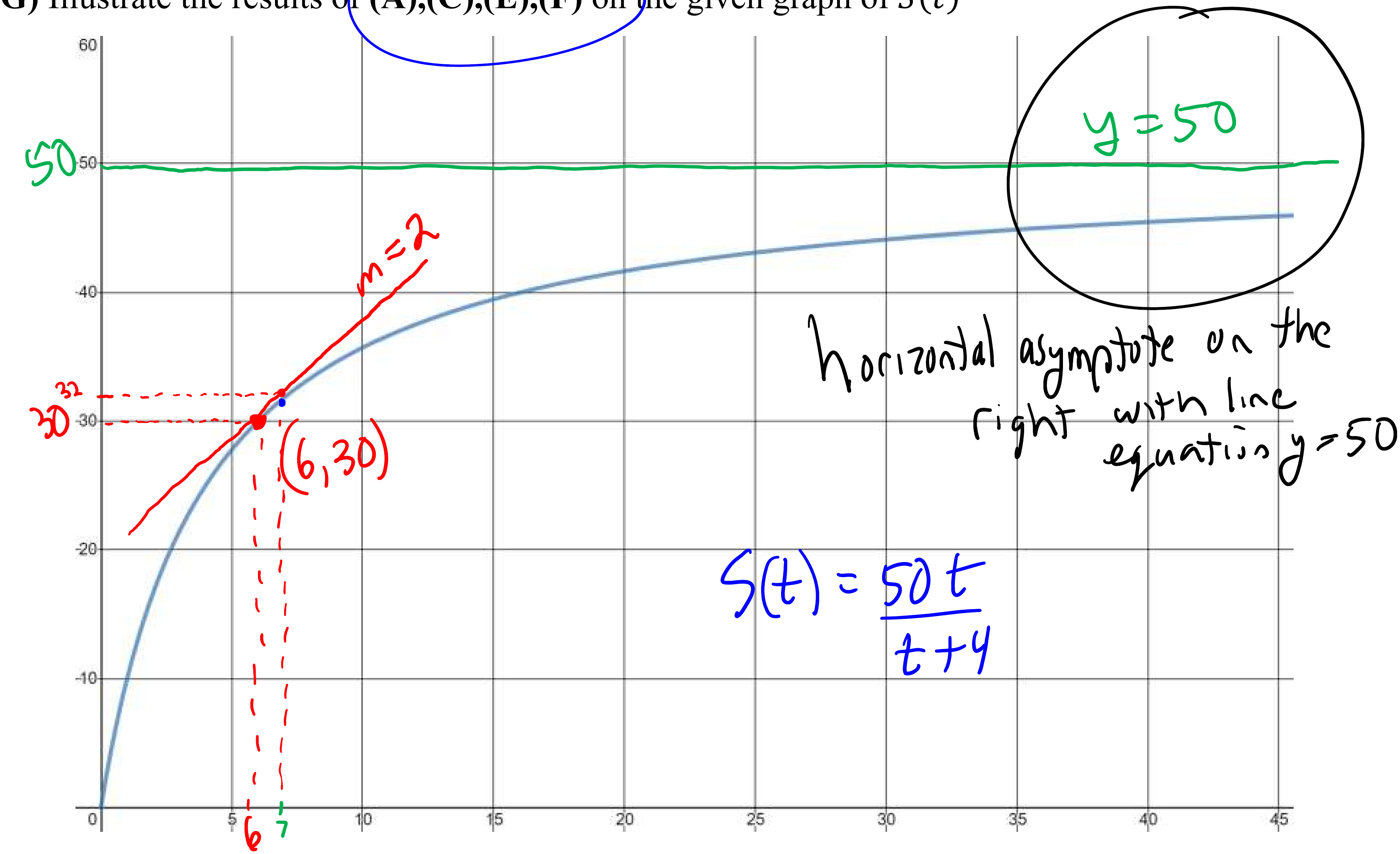
$$\lim_{t \to \infty} S(t) = \lim_{t \to \infty} \frac{50t}{t+4} = \lim_{t \to \infty} \frac{50t}{t} = \lim_{t \to \infty} 50 = 50$$

keep only the leading terms

Since $t \to \infty$, we know $t \neq 0$, so we can cancel $\frac{t}{t}$

roughly 50 thousand games will eventually sell.

(G) Illustrate the results of **(A),(C),(E),(F)** on the given graph of $S(t)$



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