Subject for this video:

Product Rule then Chain Rule

Reading:

- **General:** Section 3.4: The Chain Rule
- **More Specifically:** There is no discussion of this kind of problem in Section 3.4, and no similar examples.

Homework:

H52: Product Rule then Chain Rule (3.4#47,79)

Derivative Rules from Section 2.5 Basic Differentiation Properties

The Constant Function Rule:	$\frac{d}{dx}c = 0$
The Power Rule:	$\frac{d}{dx}x^n = nx^{n-1}$
The Sum and Constant Multiple Rule:	$\frac{d}{dx}\big(af(x) + bg(x)\big) = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$

Derivative Rules from Section 3.2 Derivatives of Exponential and Logarithmic Functions

Exponential Function Rule #1:	$\frac{d}{dx}e^{(x)} = e^{(x)}$
Exponential Function Rule #2:	$\frac{d}{dx}e^{(kx)} = ke^{(kx)}$
Exponential Function Rule #3:	$\frac{d}{dx}b^{(x)} = b^{(x)} \cdot \ln(b)$
Logarithmic Function Rule #1:	$\frac{d}{dx}\ln(x) = \frac{1}{x}$
Logarithmic Function Rule #2:	$\frac{d}{dx}\log_b(x) = \frac{1}{x\ln(b)}$

Derivative Rules from Section 3.3 Derivatives of Products and Quotients

The Product Rule:	$\frac{d}{dx} g(x) \cdot h(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$
The Quotient Rule:	$\frac{d}{dx}\left(\frac{top(x)}{bottom(x)}\right) = \frac{top'(x)bottom(x) - top(x)bottom'(x)}{\left(bottom(x)\right)^2}$

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Derivative Rule from Section 3.4 The Chain Rule

The Chain Rule:	$\frac{d}{dx} outer\left(inner(x)\right) = outer'\left(inner(x)\right) \cdot inner'(x)$

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[Example 1] (similar to 3.4#47) Let $f(x) = 7xe^{(x^2-5)}$

Find $f'(x)$

$$f'(x) = \frac{d}{dx}\left(7x\,e^{(x^2-5)}\right)$$

must use the product rule

$$= \left(\frac{d}{dx}7x\right)\cdot e^{(x^2-5)} + (7x)\frac{d}{dx}e^{(x^2-5)}$$

chain rule

$$= (7)\cdot e^{(x^2-5)} + 7x\cdot\left(e^{(x^2-5)}\cdot 2x\right)$$

$$= 7e^{(x^2-5)} + 14x^2e^{(x^2-5)}$$

$$= (7 + 14x^2)\,e^{(x^2-5)}$$

Chain Rule Details

$inner(x) = x^2-5$

$inner'(x) = 2x$

$outer(\ ) = e^{(\ )}$

empty version

$outer'(\ ) = e^{(\ )}$

[Example 2] (similar to 3.4#79) Find $\dfrac{d}{dx}[2x^2(x^3-3)^4]$

$$\frac{d}{dx}\left[2x^2\cdot(x^3-3)^4\right] = \left(\frac{d}{dx}2x^2\right)\cdot(x^3-3)^4 + 2x^2\cdot\frac{d}{dx}\left((x^3-3)^4\right)$$

↑ product rule

chain rule

$$= (2\cdot 2x)(x^3-3)^4 + 2x^2\left(4(x^3-3)^3\cdot 3x^2\right)$$

$$= 4x(x^3-3)^4 + 24x^4(x^3-3)^3$$

$$= 4x(x^3-3)^3\cdot(x^3-3) + 4x(x^3-3)^3\cdot 6x^3$$

factor

$$= 4x(x^3-3)^3\left[(x^3-3) + (6x^3)\right]$$

$$= 4x(x^3-3)^3\left[7x^3-3\right]$$

Chain Rule Details

$inner(x) = x^3-3$

$inner'(x) = 3x^2$

$outer(\ ) = (\ )^4$

empty version

$outer'(\ ) = 4(\ )^3$