Subject for this video:

Graphical Problems about Increasing and Decreasing Behavior of Functions

Reading:

- **General:** Section 4.1 First Derivative and Graphs
- **More Specifically:** In my opinion, Section 4.1 is not organized very well. The topics do not progress from simple to more complex. The exercises also do not progress from simple to more complex. Plus, the ordering of the exercises does not match the order of presentation in the reading. You may find the book a little frustrating to read. I have chosen to present concepts from Section 4.1 in an order that I feel does progress from simple to more complex. It is not possible to give guidance about what parts of Section 4.1, what examples, correspond to the topics in this video, because the topics here are scattered throughout Section 4.1

Homework:

H53: Graphical Problems about Increasing and Decreasing Behavior of Functions

(4.1#9,11,13*,14*,19,21,23,25*,61,65,83)

We reviewed the idea of *positive* and *negative* from Video for H20. It is worth recalling.

Definition of *Positive and Negative Function Values*

words: $f(c)$ *is positive*

meaning: $f(c) > 0$

behavior of graph: The point $\left(c, f(c)\right)$ is above the x axis.

words: $f(c)$ *is negative*

meaning: $f(c) < 0$

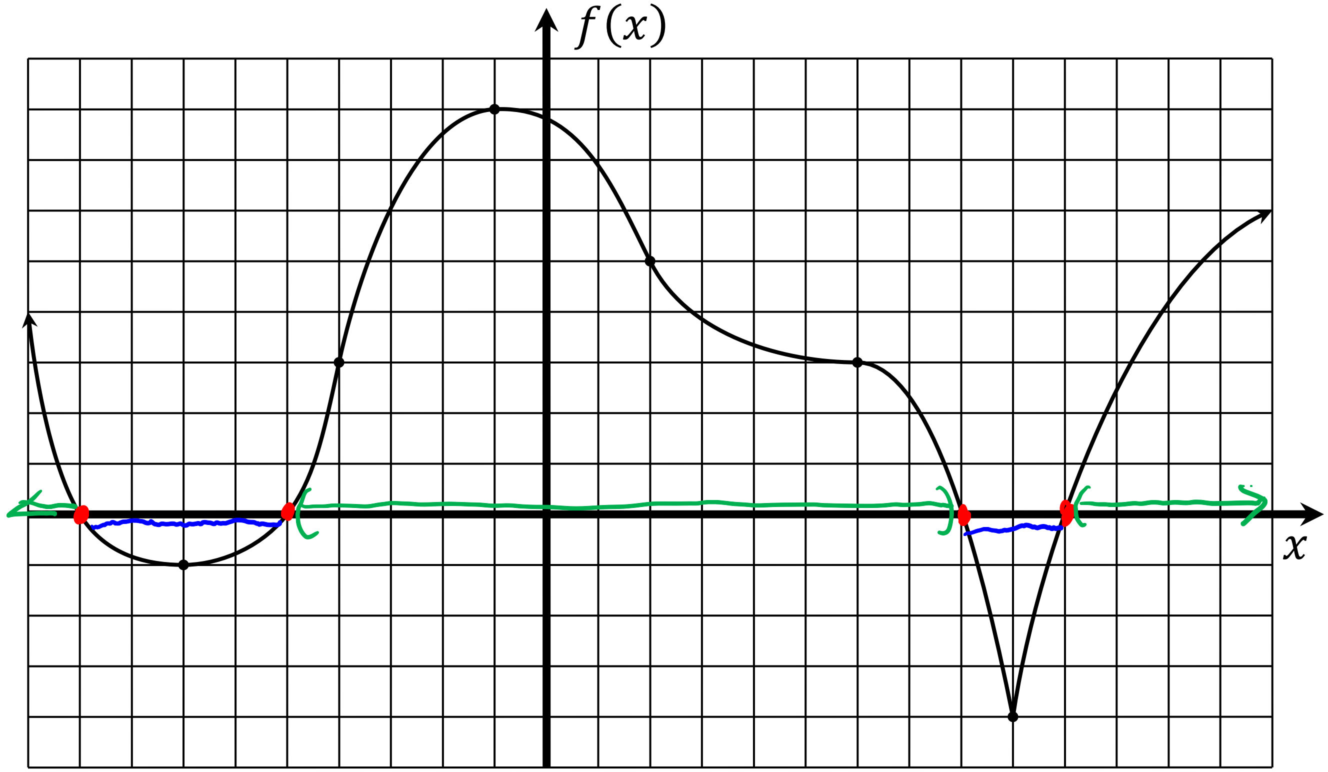
behavior of graph: The point $\left(c, f(c)\right)$ is below the x axis.

words: $f(c)$ *is zero*

meaning: $f(c) = 0$

behavior of graph: The point $\left(c, f(c)\right)$ is on the x axis.

[Example 1] (reviewing an example from Video 20) The graph of a function f is shown below.



(a) At which x values is $f(x)$ zero? $x = -9, -5, 8, 10$

(b) On what intervals is $f(x)$ positive? $(-\infty, -9), (-5, 8), (10, \infty)$

(c) On what intervals is $f(x)$ negative? $(-9, -5), (8, 10)$

A new concept is that of an increasing function.

Definition of *Increasing Function Values on an Interval*

words: $f(x)$ *is increasing on the interval* (a, b)

meaning: If $a < x_1 < x_2 < b$ then $f(x_1) < f(x_2)$.

behavior of graph: As you move from left to right in the interval (a, b) the graph of $f(x)$ goes *up*.

words: $f(x)$ *is decreasing on the interval* (a, b)

meaning: If $a < x_1 < x_2 < b$ then $f(x_1) > f(x_2)$.

behavior of graph: As you move from left to right in the interval (a, b) the graph of $f(x)$ goes *down*.

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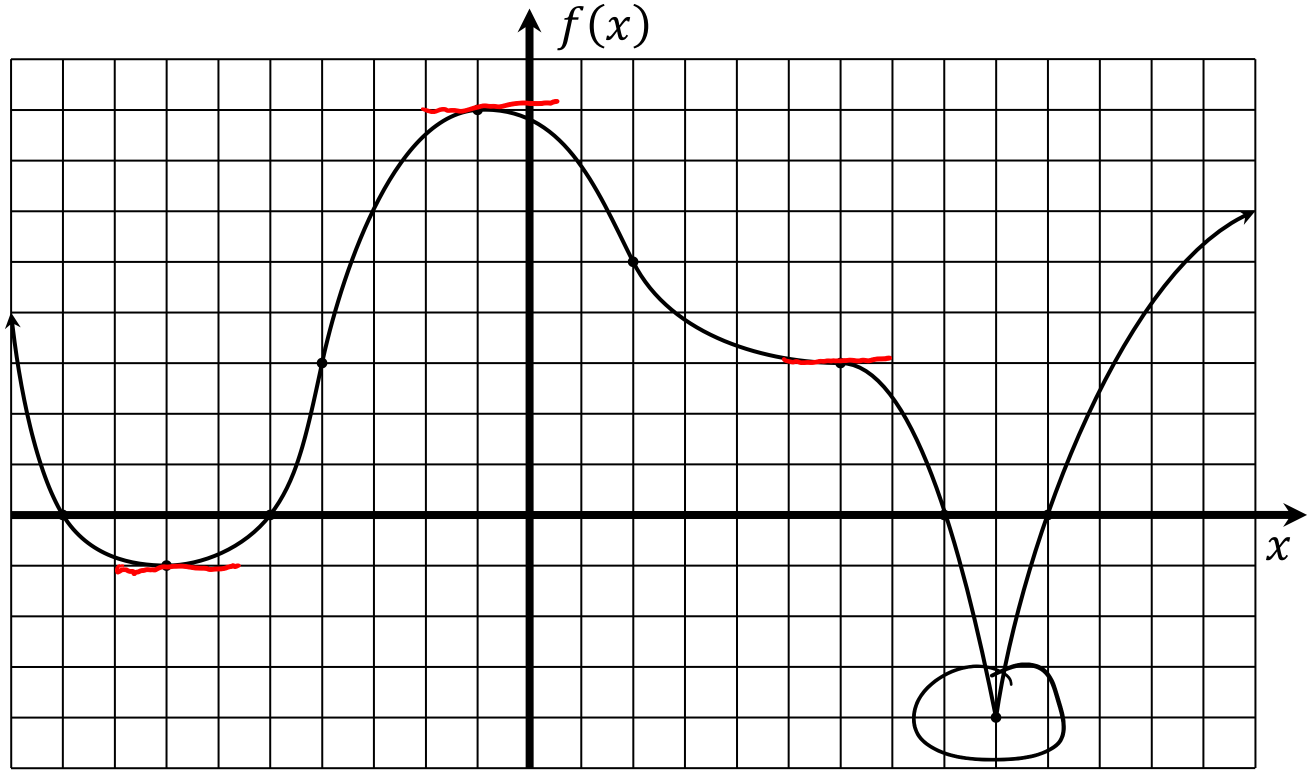
Resume [Example 1]

(similar to 4.1#9)



(d) At which x values is the line tangent to the graph of $f(x)$ horizontal? $x = -7, -1, 6$

(e) At which x values does the graph of $f(x)$ not have a tangent line? $x = 9$

(f) On what intervals is $f(x)$ increasing? $(-7, -1), (9, \infty)$

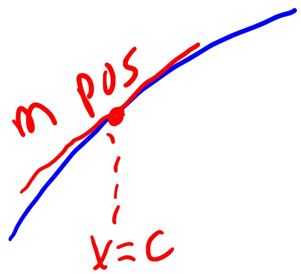
$x = -7, \quad x = -1$

(g) On what intervals is $f(x)$ decreasing? $(-\infty, -7), (-1, 9)$

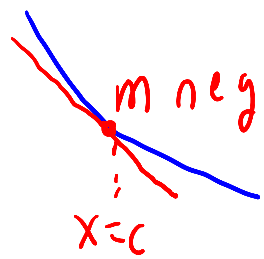
Correspondence between

sign behavior of $f'(x)$ at a particular $x = c$ and behavior of the graph of $f(x)$ at $x = c$

- If $f'(c)$ is positive then the line tangent to graph of $f(x)$ at $x = c$ tilts upward



- If $f'(c)$ is negative then the line tangent to graph of $f(x)$ at $x = c$ tilts downward



- If $f'(c)$ is ~~negative~~ zero then the line tangent to graph of $f(x)$ at $x = c$ is horizontal

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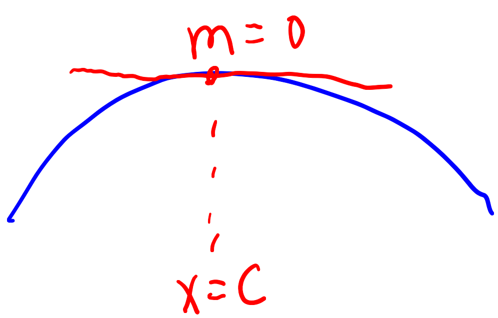


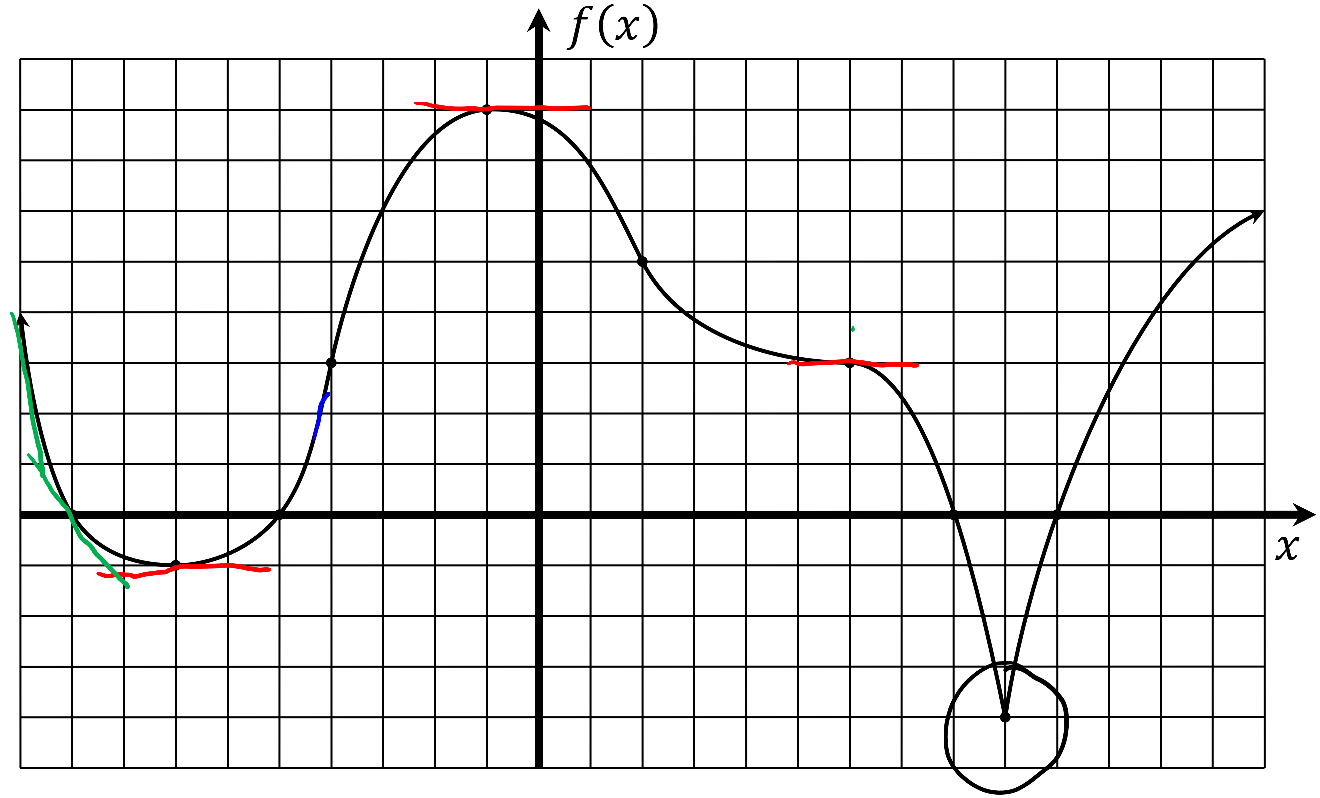
Correspondence between

sign behavior of $f'(x)$ on an interval (a, b) and behavior of the graph of $f(x)$ on the interval (a, b)

- If $f'(x)$ is positive on an interval (a, b) then $f(x)$ is increasing on the interval (a, b).
- If $f'(x)$ is negative on an interval (a, b) then $f(x)$ is decreasing on the interval (a, b).
- If $f'(x)$ is zero on an interval (a, b) then $f(x)$ is constant on the interval (a, b).

Resume [Example 1]

(similar to 4.1#11)



(h) At which x values is $f'(x)$ zero? $x = -7, -1, 6$

(i) At which x values does $f'(x)$ not exist? $x = 9$ because no tangent line

(j) On what intervals is $f'(x)$ positive? $(-7, -1), (9, \infty)$

(k) On what intervals is $f'(x)$ negative? $(-\infty, -7), (-1, 6), (6, 9)$

Notice that the answers to **(g)** and **(k)** are not the same. There is some subtlety here.

Remember that we learned that

If $f'(x)$ is negative on an interval (a, b)

then $f(x)$ is guaranteed to be decreasing on the interval (a, b).

But this does not work the other way. That is,

If $f(x)$ is decreasing on the interval (a, b)

we are ***not*** guaranteed that $f'(x)$ will be negative on an interval (a, b)

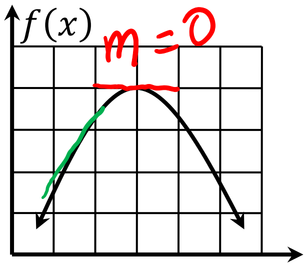
The answers to **(g)** and **(k)** are a perfect illustration of this.

The function $f(x)$ is decreasing on the whole interval $(-1,9)$

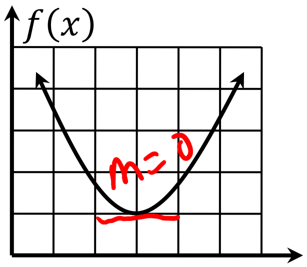
But $f'(x)$ is not negative on the whole interval $(-1,9)$.

$f'(x)$ is negative on $(-1,6)$ and $(6,9)$.

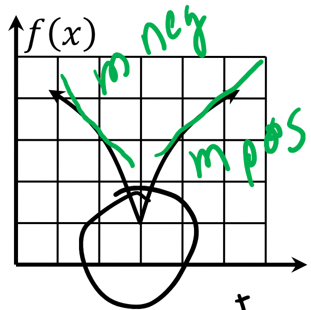
[Example 2] (Similar to 4.1#19,21,23) Match the graph of $f(x)$ to the sign chart for $f'(x)$



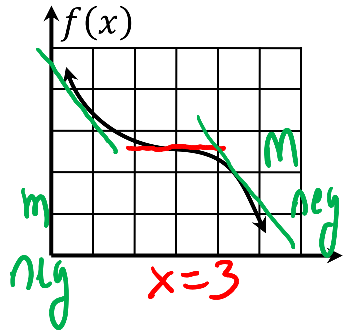
x=3



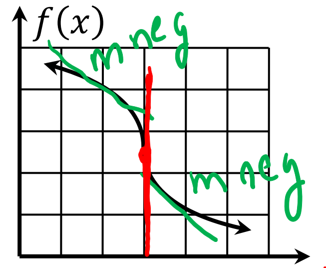
x=3



no tangent at x=3



x=3



vertical tangent at x=3

Slope is not defined

$f'(x)$: --------- 0 +++++ ; $x = 3$

$f'(x)$: ------- ND ------- ; $x = 3$

derivative not defined at x=3

$f'(x)$: ------- 0 ------- ; $x = 3$

$f'(x)$: +++++ 0 -------- ; $x = 3$

$f'(x)$: -------- ND +++++ ; $x = 3$

derivative not defined at x=3

[Example 3] (similar to 4.1#61) Given this information

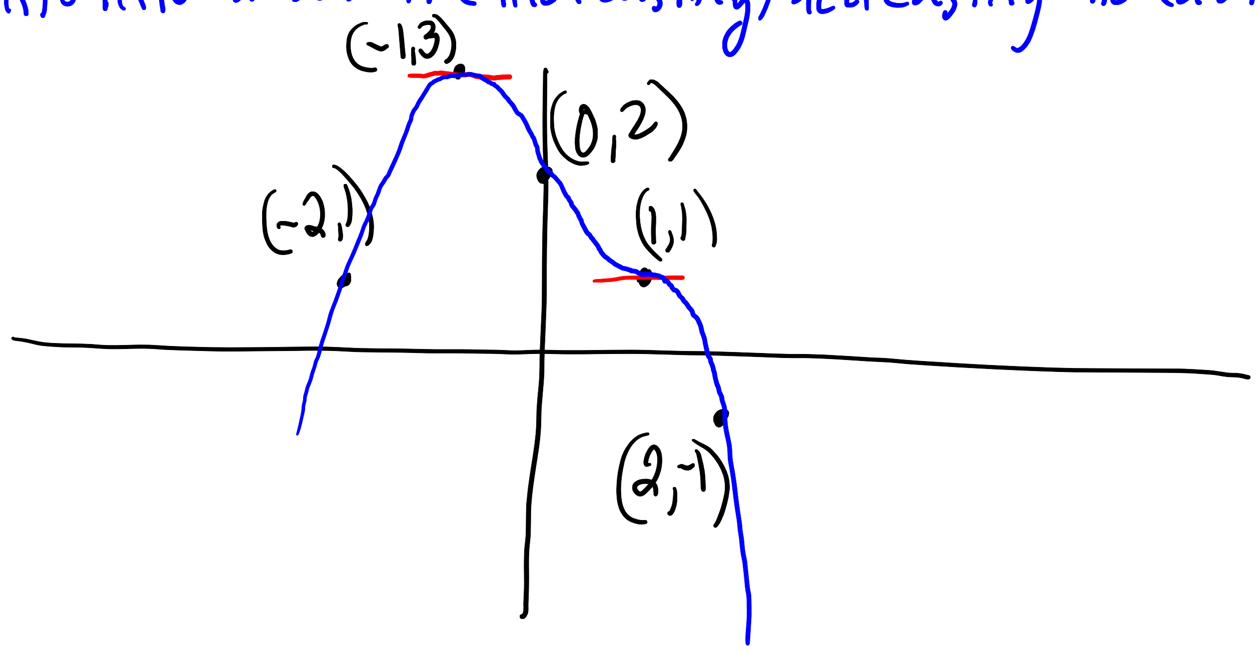
$f(x)$ is continuous for $(-\infty, \infty)$.

x	-2	-1	0	1	2
$f(x)$	1	3	2	1	-1

$f'(x)$

$+ + +$	0	$- - -$	0	$- - -$	x
f increasing	-1	f decreasing	1	f decreasing	
	f horiz		f horiz		

Sketch the graph of f. Start by translating information about sign of $f'(x)$ into info about the increasing/decreasing behavior of f



[Example 4] (similar to 4.1#65) Given this information

$f(x)$ is continuous for $(-\infty, \infty)$

$f(-1) = 2, \qquad f(0) = 0, \qquad f(1) = -2$

$f'(-1) = 0, \qquad f'(0)\ is\ undefined, \qquad f'(1) = 0$

$f'(x) > 0$ on $(-\infty, -1)\ and\ (1, \infty)$

$f'(x) < 0$ on $(-1,0)\ and\ (0,1)$

Sketch the graph of f.

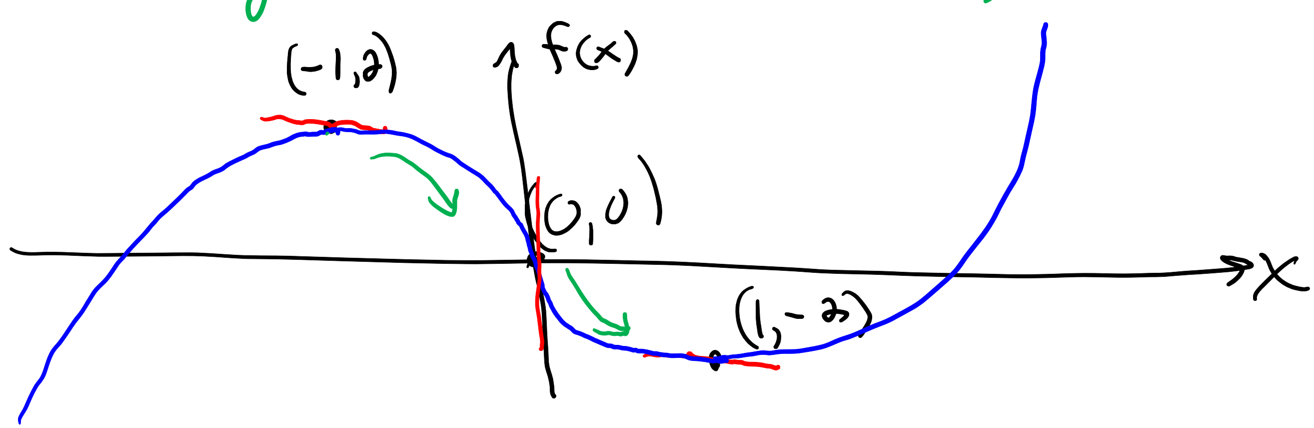
Translate info about sign behavior of $f'(x)$ into info about increasing/decreasing behavior of $f(x)$

$f(x)$ has horiz tangents at $x=-1$ and $x=1$

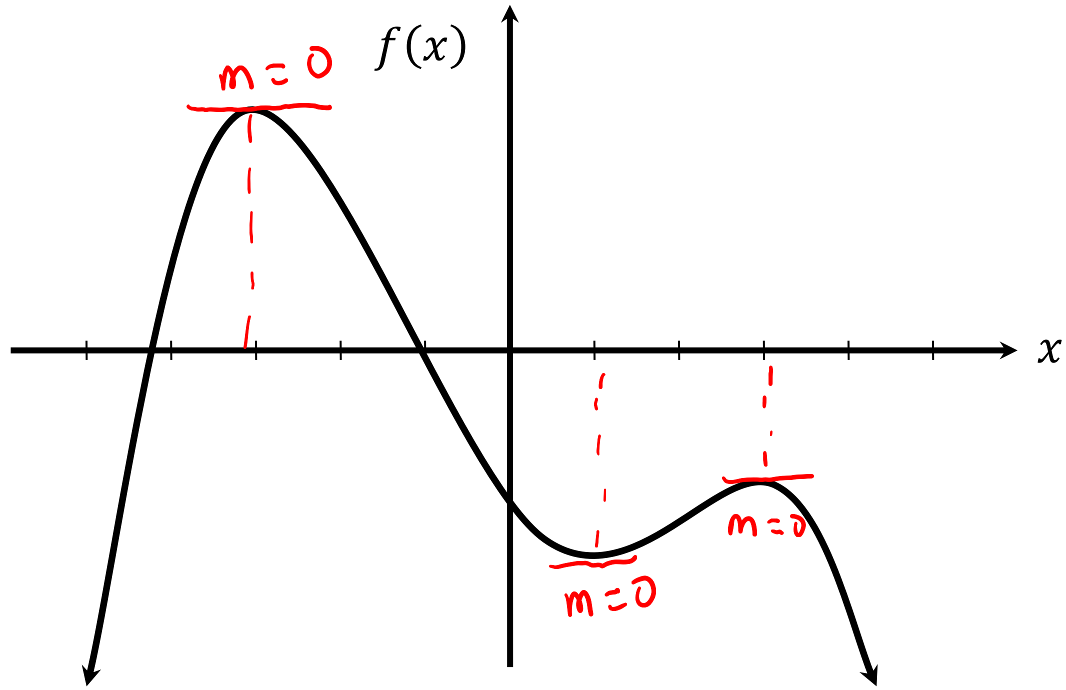
$f(x)$ either has no tangent line at $x=0$ or a vertical tangent

$f(x)$ increasing on $(-\infty,-1)$ and $(1,\infty)$

$f(x)$ decreasing on $(-1,0)$ and $(0,1)$



[Example 5] (similar to 4.1#83)



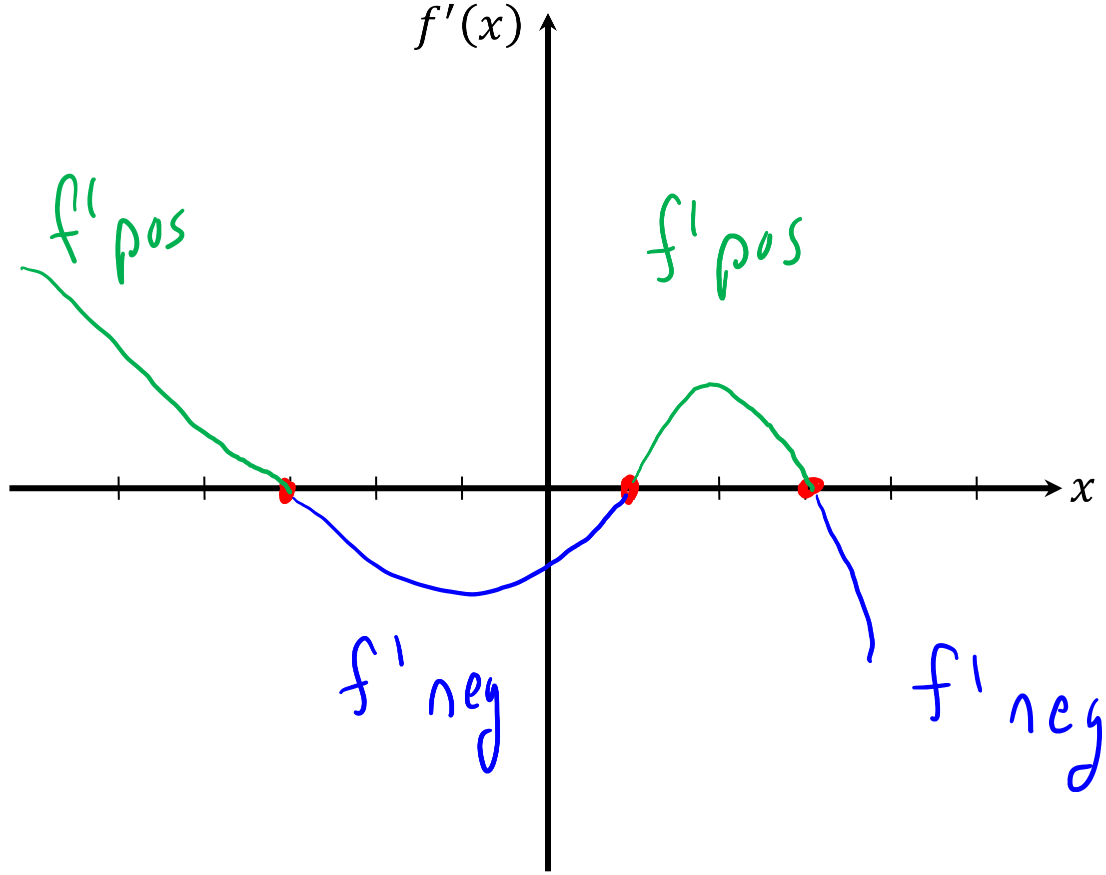
Use the given graph of $y = f(x)$ to answer the following questions:

(A) Find the intervals on which $f'(x) > 0$. $(-\infty, -3), (1, 3)$

(B) Find the intervals on which $f'(x) < 0$. $(-3, 1), (3, \infty$

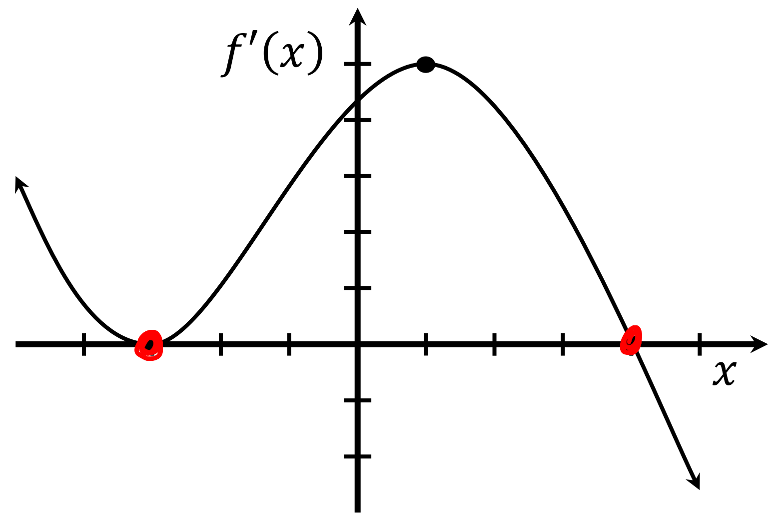
(C) Find the values of x for which $f'(x) = 0$. $x = -3, 1, 3$

Then sketch a possible graph of $y = f'(x)$.



[Example 6]

The graph of f' is shown at right.



(A) Fill in the table below.

x	sign of $f'(x)$ (circle one)	conclusions about behavior of function $f(x)$
$x < -3$	pos neg zero	f is increasing
$x = -3$	pos neg zero	f has horiz Tangent
$-3 < x < 1$	pos neg zero	
$x = 1$	pos neg zero	f is increasing
$1 < x < 4$	pos neg zero	
$x = 4$	pos neg zero	f has horiz tangent
$4 < x$	pos neg zero	f decreasing

(B) Sketch a possible graph of $f(x)$ below.