Subject for this video:

Increasing and Decreasing Intervals for a Function Given by a Formula

Reading:

- **General:** Section 4.1 First Derivative and Graphs
- **More Specifically:** In my opinion, Section 4.1 is not organized very well. The topics do not progress from simple to more complex. The exercises also do not progress from simple to more complex. Plus, the ordering of the exercises does not match the order of presentation in the reading. You may find the book a little frustrating to read. I have chosen to present concepts from Section 4.1 in an order that I feel does progress from simple to more complex. It is not possible to give guidance about what parts of Section 4.1, what examples, correspond to the topics in this video, because the topics here are scattered throughout Section 4.1

Homework:

H54: Increasing and Decreasing Intervals for a Function Given by a Formula

(4.1#49,51*,53,55))

We will use these Important Tools from Section 2.3 Continuity: Sign Charts

THEOREM 2 Sign Properties on an Interval (a, b)

If f is continuous on (a, b) and $f(x) \neq 0$ for all x in (a, b), then either $f(x) > 0$ for all x in (a, b) or $f(x) < 0$ for all x in (a, b).

DEFINITION

A real number x is a **partition number** for a function f if f is discontinuous at x or $f(x) = 0$.

PROCEDURE Constructing Sign Charts

Given a function f,

Step 1 Find all partition numbers of f:

(A) Find all numbers x such that f is discontinuous at x. (Rational functions are discontinuous at values of x that make a denominator 0.)

(B) Find all numbers x such that $f(x) = 0$. (For a rational function, this occurs where the numerator is 0 and the denominator is not 0.)

Step 2 Plot the numbers found in step 1 on a real number line, dividing the number line into intervals.

Step 3 Select a test number in each open interval determined in step 2 and evaluate $f(x)$ at each test number to determine whether $f(x)$ is positive $(+)$ or negative $(-)$ in each interval.

Step 4 Construct a sign chart, using the real number line in step 2. This will show the sign of $f(x)$ on each open interval.

And we will use these Important Tools from the previous Video (Section 4.1 concepts):

Correspondence between

sign behavior of $f'(x)$ at a particular $x = c$ and behavior of the graph of $f(x)$ at $x = c$

- If $f'(c)$ is positive then the line tangent to graph of $f(x)$ at $x = c$ tilts upward
- If $f'(c)$ is negative then the line tangent to graph of $f(x)$ at $x = c$ tilts downward
- If $f'(c)$ is negative then the line tangent to graph of $f(x)$ at $x = c$ is horizontal

Correspondence between

sign behavior of $f'(x)$ on an interval (a, b) and behavior of the graph of $f(x)$ on the interval (a, b)

- If $f'(x)$ is positive on an interval (a, b) then $f(x)$ is increasing on the interval (a, b).
- If $f'(x)$ is negative on an interval (a, b) then $f(x)$ is decreasing on the interval (a, b).
- If $f'(x)$ is zero on an interval (a, b) then $f(x)$ is constant on the interval (a, b).

We will do two examples today. They have different functions but the same tasks:

[Example 1] (similar to Exercise 4.1#49,51,53,55) $f(x) = -x^3 + 3x^2 + 9x + 5$.

[Example 2] (similar to Exercise 4.1#49,51,53,55) $f(x) = -x^4 + 4x^3$.

(a) Find intervals on which $f(x)$ is positive or negative

(b) Find intervals on which $f(x)$ is increasing or decreasing

(c) Sketch graph, labeling all important features

- Add horizontal tangent lines
- Put (x, y) coordinates on all prominent points
  - Axis intercepts
  - Points where there is a horizontal tangent line

[Example 1] (similar to Exercise 4.1#49,51,53,55) $f(x) = -x^3 + 3x^2 + 9x + 5$.

(a) Find intervals on which $f(x)$ is positive or negative

Solution
- Find partition numbers for $f(x)$
- make a sign chart for $f(x)$
- use the sign chart to answer the question

Partition numbers

$f(x)$ is polynomial, so always continuous.
The only partition numbers will be the x values that cause $f(x)=0$.

Set $f(x)=0$ and solve for x

$$0 = f(x) = -x^3 + 3x^2 + 9x + 5 = -(x+1)^2(x-5)$$

factor

check: $-(x+1)^2(x-5) = \cdots$ multiply out $\cdots = -x^3 + 3x^2 + 9x + 5$

Solutions $x=-1$, $x=5$ these are the partition numbers for $f(x)$

Sign Chart for $f(x)$

f pos | $f(-1)=0$ | f pos | $f(5)=0$ | f neg

test $x=-2$ | $x=-1$ | test $x=0$ | $x=5$ | test $x=6$

$$f(-2) = -((-2)+1)^2 \cdot ((-2)-5) = -(-1)^2 \cdot (-7) = -(1)(-7) = \text{pos}$$
$$f(0) = -((0)+1)^2 \cdot ((0)-5) = -(1)^2 \cdot (-5) = -(1)(-5) = \text{pos}$$
$$f(6) = -((6)+1)^2 \cdot ((6)-5) = -(7)^2 \cdot (1) = -(49)(1) = \text{neg}$$

$f(x)$ is positive on the intervals $(-\infty, -1)$ and $(-1, 5)$

$x=-1$ $x=5$

$f(x)$ is negative on the interval $(5, \infty)$

answer to (a)

(b) Find intervals on which $f(x)$ is increasing or decreasing

Recall if $f'(x)$ is positive on an interval (a,b) then $f(x)$ is increasing on the interval (a,b)

" " " negative " " " " " " " decreasing " " " "

Strategy
- find $f'(x)$
- determine the sign behavior of $f'(x)$ using a sign chart
- use the sign chart for $f'(x)$ to make conclusions about the increasing + decreasing behavior of $f(x)$.

Find $f'(x)$

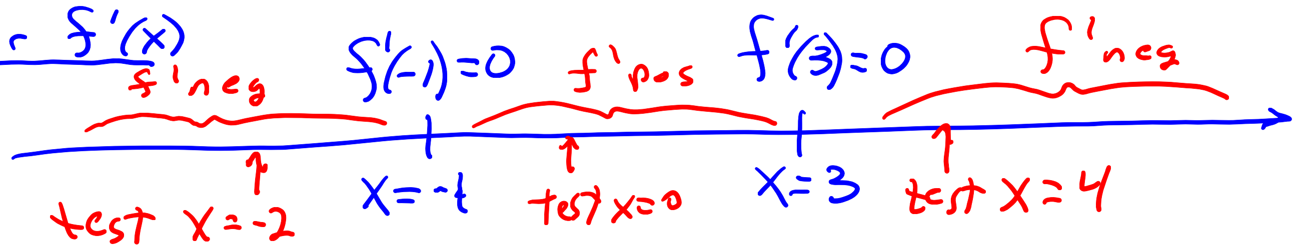
$$f'(x) = \frac{d}{dx}\left(-x^3+3x^2+9x+5\right) = -3x^2+6x+9$$

Find partition numbers for $f'(x)$
- $f'(x)$ is a polynomial, so it is always continuous.
- The only partition numbers will be the x-values that cause $f'(x)=0$.

$$0 = f'(x) = -3x^2+6x+9 = -3(x^2-2x-3) = -3(x+1)(x-3)$$

The solutions are $x=-1$ and $x=3$. These are the partition numbers for $f'(x)$

Signchart for $f'(x)$



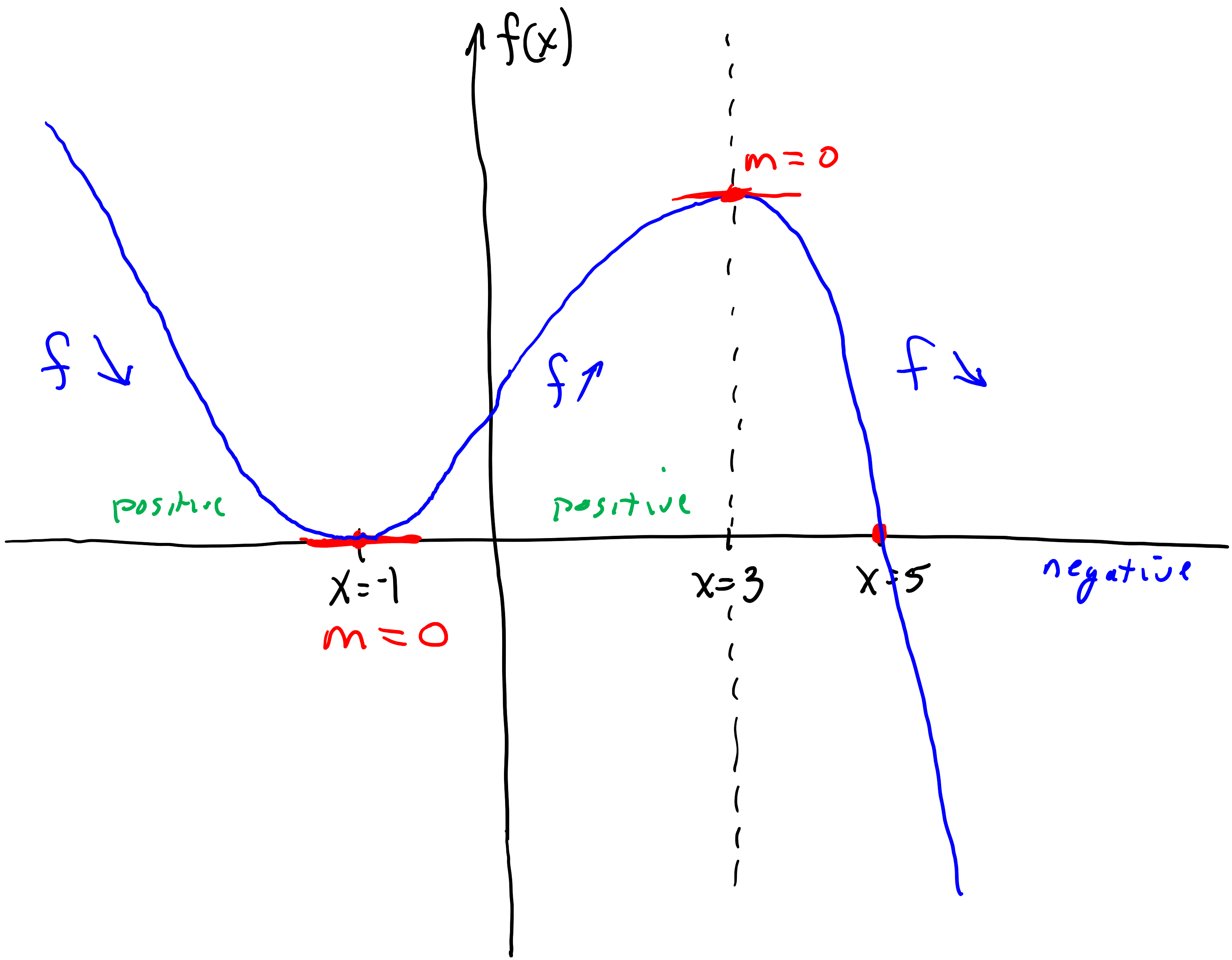
$$f'(-2) = -3((-2)+1)((-2)-3) = -3(-1)(-5) = \text{neg}$$
$$f'(0) = -3((0)+1)((0)-3) = -3(1)(-3) = \text{pos}$$
$$f'(4) = -3((4)+1)((4)-3) = -3(5)(1) = \text{neg}$$

$f(x)$ is decreasing on the intervals $(-\infty, -1)$ and $(3, \infty)$ because f' is negative there.

$f(x)$ is increasing on the interval $(-1, 3)$ because $f'(x)$ is positive

answer to (b)

(c) Sketch graph, labeling all important features



[Example 2] (similar to Exercise 4.1#49,51,53,55) $f(x) = -x^4 + 4x^3$.

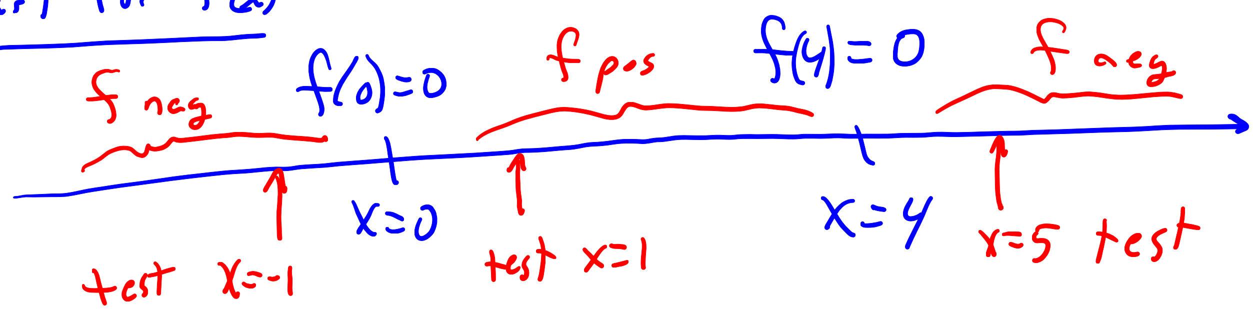
(a) Find intervals on which $f(x)$ is positive or negative

find partition numbers for $f(x)$

$$0 = f(x) = \underbrace{-x^4 + 4x^3}_{\text{standard form}} = \underbrace{-x^3(x-4)}_{\text{factored form}}$$

Solutions are $x=0,\ x=4$. These are the partition numbers for $f(x)$.

Make sign chart for $f(x)$



$$f(-1) = -(-1)^3((-1)-4) = -(-1)(-5) = \text{neg}$$

$$f(1) = -(1)^3((1)-4) = -(1)(-3) = \text{pos}$$

$$f(5) = -(5)^3((5)-4) = -(125)(1) = \text{neg}$$

$f(x)$ is positive on the interval $(0,4)$

$f(x)$ is negative on the intervals $(-\infty, 0)$, $(4, \infty)$

(b) Find intervals on which $f(x)$ is increasing or decreasing

Strategy:
- find $f'(x)$
- make sign chart for $f'(x)$
- use info about sign behavior of $f'(x)$ to determine increasing & decreasing behavior of $f(x)$.

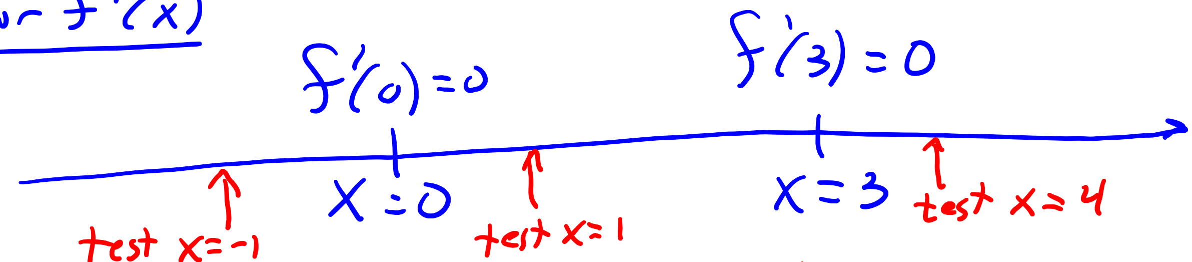
$$f'(x) = \frac{d}{dx}\left(-x^4 + 4x^3\right) = -4x^3 + 12x^2$$

The only partition numbers for $f'(x)$ will be the x values that cause $f'(x) = 0$

$$0 = f'(x) = \underbrace{-4x^3 + 12x^2}_{\text{standard form}} = \underbrace{-4x^2(x-3)}_{\text{factored form}}$$

The solutions are $x=0$ and $x=3$. These are the partition numbers for $f'(x)$

Sign chart for $f'(x)$



$$f'(-1) = -4(-1)^2((-1)-3) = -4(1)(-4) = \text{pos}$$

$$f'(1) = -4(1)^2((1)-3) = -4(1)(-2) = \text{pos}$$

$$f'(4) = -4(4)^2((4)-3) = -4(16)(1) = \text{neg}$$

$f(x)$ is increasing on the intervals $(-\infty, 0)$ and $(0, 3)$ because $f'(x)$ is positive

$f(x)$ is decreasing on the interval $(3, \infty)$ because $f'(x)$ is negative.

(c) Sketch graph, labeling all important features

Observe:

f(x) is actually increasing on the whole interval $(-\infty, 3)$!!