Subject for this video:

Partition Numbers for $f'(x)$; Critical Numbers for $f(x)$

Reading:

- **General:** Section 4.1 First Derivative and Graphs
- **More Specifically:** In my opinion, Section 4.1 is not organized very well. The topics do not progress from simple to more complex. The exercises also do not progress from simple to more complex. Plus, the ordering of the exercises does not match the order of presentation in the reading. You may find the book a little frustrating to read. I have chosen to present concepts from Section 4.1 in an order that I feel does progress from simple to more complex. It is not possible to give guidance about what parts of Section 4.1, what examples, correspond to the topics in this video, because the topics here are scattered throughout Section 4.1

Homework:

H55: Partition Numbers for $f'(x)$; Critical Numbers for $f(x)$ (4.1#27,29,31*)

Recall the definition of Continuity from Section 2.3

Definition of *Continuity at a particular x value*

Words: *The function f is continuous at $x = c$.*

Meaning: the function f passes these three tests:

Test 1: $\lim_{x \to c} f(x)$ exists

Test 1a: $\lim_{x \to c^-} f(x)$ exists

Test 1b: $\lim_{x \to c^+} f(x)$ exists

Test 1c: The numbers in test 1a and 1b agree.

Test 2: $f(c)$ exists

Test 3: The numbers in *Test 1* and *Test 2* agree.

A function fails to be continuous if it fails any of the three tests.

And recall the definition of *partition number*, also from Section 2.3

> **DEFINITION**
>
> A real number x is a **partition number** for a function f if f is discontinuous at x or $f(x) = 0$.

We have seen that partition numbers for a function are important, because they are the only x values where the function $f(x)$ can *change sign*.

We see that in order to find the partition numbers for a function $f(x)$, we have to find

- the x values that cause $f(x) = 0$
- the x values where $f(x)$ is discontinuous (by failing any of the three continuity tests)

Partition Numbers for $f'(x)$

In Section 4.1, we have seen that it is useful to determine the sign behavior of $f'(x)$, because information about the sign behavior of $f'(x)$ can be translated into information about the increasing and decreasing behavior of ~~$f'(x)$.~~ $f(x)$

In order to determine the sign behavior of $f'(x)$, we make a sign chart for $f'(x)$.

And in order to make a sign chart for $f'(x)$, we start by finding the *partition numbers* for $f'(x)$.

And in order to find the partition numbers for $f'(x)$, we have to find

- the x values that cause $f'(x) = 0$
- the x values where $f'(x)$ is discontinuous (by failing any of the three continuity tests)

It turns out that for the functions that we study in this course, the job of finding the partition numbers for $f'(x)$ is made slightly simpler because of the kinds of functions we study: In this course, we only work with functions $f(x)$ whose derivatives $f'(x)$ only fail Continuity Test 1 at x values where they also fail Continuity Test 2. That means that in our course, the only x values where $f'(x)$ will *fail to be continuous* will be the x values where $f'(x)$ *fails to exist*.

This allows us to give a simpler definition for a partition number for $f'(x)$.

Definition of *Partition Number for* $\boldsymbol{f'(x)}$

Words: *partition number for* $f'(x)$

Meaning: a number $x = c$ such that $f'(c) = 0$ or $f'(c)$ *does not exist*

Before going on, remember why we are interested in the partition numbers for $f'(x)$

- The partition numbers for $f'(x)$ are the only x values where $f'(x)$ can *change sign*.
- On the intervals between the partition numbers for $f'(x)$, the sign of $f'(x)$ does not change, and so the increasing/decreasing behavior of $f(x)$ also does not change.

Critical Numbers for $f(x)$

We were led to think about partition numbers for $f'(x)$ by wanting to find the intervals where $f(x)$ is increasing or decreasing. Those will always be intervals between the partition numbers for $f'(x)$.

It turns out that it is useful to scrutinize the partition numbers for $f'(x)$ more carefully. We will be interested in finding out which partition numbers for $f'(x)$ also satisfy an additional requirement. Those that *do* satisfy the additional requirement will be called *critical numbers* for $f(x)$.

Definition of *Critical Number for* $f(x)$

Words: *critical number for* $f(x)$

Meaning: a number $x = c$ that satisfies these two requirements:

- The number $x = c$ is a partition number for $f'(x)$.
- The number $x = c$ is in the domain of $f(x)$.

That is,

- $f'(c) = 0$ or $f'(c)$ does not exist
- $f(c)$ exists

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We will do three examples today. They have different functions but the same tasks:

[Example 1] (similar to 4.1#27) $f(x) = x^3 - 75x + 57$.

[Example 2] (similar to 4.1#29) $f(x) = \frac{1}{x-5}$

[Example 3] (similar to 4.1#31) $f(x) = x^{1/5}$.

(A) Find $f'(x)$

(B) Find the *partition numbers for* $f'(x)$.

(C) Find the *critical numbers for* $f(x)$.

[Example 1] (similar to 4.1#27) $f(x) = x^3 - 75x + 57$.

(A) Find $f'(x)$

(B) Find the *partition numbers for* $f'(x)$.

(C) Find the *critical numbers for* $f(x)$.

(A) $f'(x) = \frac{d}{dx}\left(x^3 - 75x + 57\right) = 3x^2 - 75 = 3(x^2-25) = 3(x+5)(x-5)$

($3x^2 - 75$: standard form; $3(x+5)(x-5)$: factored form)

(B) $f'(x)$ is a polynomial. So there are no x values that cause $f'(x)$ to not exist.

So the only partition numbers for $f'(x)$ will be the solutions to the equation $f'(x) = 0$

$$3(x+5)(x-5) = 0$$

Solutions: $x = -5$, $x = 5$ Partition numbers for $f'(x)$

(C) The critical numbers for $f(x)$ must be

- partition numbers for $f'(x)$.
- must also be numbers in the domain of $f(x)$.

But $f(x)$ is a polynomial. Its domain is the set of all real numbers

So the numbers $x = -5$ and $x = 5$ are also critical numbers for $f(x)$

[Example 2] (similar to 4.1#29) $f(x) = \dfrac{1}{x-5}$

(A) Find $f'(x)$

(B) Find the *partition numbers for* $f'(x)$.

(C) Find the *critical numbers for* $f(x)$.

(A) Start by rewriting $f(x) = \dfrac{1}{X-5} = (X-5)^{-1}$

Then $f'(X) = \dfrac{d}{dx}(x-5)^{-1}$

$= \dfrac{d}{dx}\,\text{outer}(\text{inner}(x))$

chain rule

$= \text{outer}'(\text{inner}(x)) \cdot \text{inner}'(x)$

$= -\dfrac{1}{(X-5)^2} \cdot (1)$

$= -\dfrac{1}{(X-5)^2}$

Chain Rule Details

$\text{inner}(x) = X-5$

$\text{inner}'(x) = \dfrac{d}{dx}(x-5) = 1-0 = 1$

$\text{outer}(\ ) = (\ )^{-1}$ power function

$\text{outer}'(\ ) = (-1)(\ )^{-1-1}$

$= (-1)(\ )^{-2}$

$= (-1) \cdot \dfrac{1}{(\ )^2}$

$= -\dfrac{1}{(\ )^2}$

Positive exponent form

(B) Partition numbers for $f'(x) = \dfrac{-1}{(x-5)^2}$

Notice that $f'(5) = \dfrac{-1}{((5)-5)^2} = \dfrac{-1}{0}$ Does not exist.

So $x=5$ is a partition number for $f'(x)$.

Are there any x values that will cause $f'(x)=0$?

Notice that $f'(x) = \dfrac{-1}{(x-5)^2}$ is a fraction,

and remember that a fraction $\frac{a}{b}=0$ only when $a=0$ and $b\neq 0$.

But $f'(x)$ has numerator $a=-1$. It will never be zero.

So there are no x values that will cause $f'(x)=0$.

So the only partition number for $f'(x)$ is $x=5$.

(C) The only possible critical number for $f(x)$ would be $x=5$

But $x=5$ is not in the domain of $f(x) = \dfrac{1}{x-5}$.

Conclude that $f(x)$ has no critical numbers!

[Example 3] (similar to 4.1#31) $f(x) = x^{1/5}$.

(A) Find $f'(x)$

(B) Find the *partition numbers for* $f'(x)$.

(C) Find the *critical numbers for* $f(x)$.

(A) $f'(x) = \frac{d}{dx} x^{\frac{1}{5}} = \left(\frac{1}{5}\right) \cdot x^{\frac{1}{5}-1} = \left(\frac{1}{5}\right) x^{-\frac{4}{5}} = \left(\frac{1}{5}\right) \cdot \frac{1}{x^{4/5}} = \frac{1}{5x^{4/5}}$

↑ power rule with $n = \frac{1}{5}$

convert to positive exponent form

(B) Partition numbers for $f'(x)$

Notice that $f'(x) = \frac{1}{5x^{4/5}}$ is a fraction whose numerator will never be zero. So $f'(x)$ will never be zero.

Observe that $f'(0) = \frac{1}{5(0)^{4/5}} = \frac{1}{5 \cdot 0} = \frac{1}{0}$ does not exist.

So $x=0$ is the only partition number for $f'(x)$

(C) Critical numbers for $f(x)$

See if $x=0$ is in the domain of $f(x)$

$f(0) = 0^{1/3} = 0$ this does exist. So $x=0$ is in domain of $f(x)$.

So $x=0$ is a critical number for $f(x)$