Subject for this video:

Finding Second Derivatives

Reading:

- **General:** Section 4.2 Second Derivatives and Graphs
- **More Specifically:** The concepts in this video are scattered throughout Section 4.2 of the book. There are no book examples similar to the examples in this video.

Homework:

H58: Finding Second Derivatives (4.2#17,19)

From the Previous Video

Definition of the Second Derivcative

Words: *the second derivative of* $f(x)$.

Symbols: $f''(x), \quad \frac{d^2}{dx^2}f(x)$

Meaning: $f''(x) = \frac{d}{dx}f'(x) = \frac{d}{dx}\frac{d}{dx}f(x)$

[Example 1] Find the second derivative of $f(x) = -x^3 + 2x^2 - 3x + 9$.

Solution

$f'(x) = \frac{d}{dx}(-x^3 + 2x^2 - 3x + 9) = -3x^2 + 4x - 3$

$f''(x) = \frac{d}{dx}f'(x) = \frac{d}{dx}(-3x^2 + 4x - 3) = -6x + 4$

[Example 2] Find $g''(x)$ for $g(x) = -6x^{-2} + 12x^{-3}$. power function form

Solution

$$g'(x) = \frac{d}{dx}\left(-6x^{-2} + 12x^{-3}\right) = -6(-2)x^{-2-1} + 12(-3)x^{-3-1}$$

$$= 12x^{-3} - 36x^{-4}$$

$$g''(x) = \frac{d}{dx}g'(x) = \frac{d}{dx}\left(12x^{-3} - 36x^{-4}\right) =$$

$$= 12(-3)x^{-3-1} - 36(-4)x^{-4-1}$$

$$= -36x^{-4} + 144x^{-5}$$ power function form

$$= \frac{-36}{x^4} + \frac{144}{x^5}$$ positive exponent form.

[Example 3] find $\frac{d^2y}{dx^2}$ for $y = -\frac{7}{x^5} + \frac{12}{\sqrt{x}}$ Positive exponent form and radical form

Start by converting to power function form

$y = -\frac{7}{x^5} + \frac{12}{\sqrt{x}} = -7 \cdot \frac{1}{x^5} + 12 \cdot \frac{1}{x^{1/2}} = -7x^{-5} + 12x^{-1/2}$ Power function form

↑ Separate the constants

↑ Convert to power functions

Now find the derivative

$\frac{dy}{dx} = \frac{d}{dx}\left(-7x^{-5} + 12x^{-1/2}\right) = -7(-5)x^{-5-1} + 12\left(\frac{-1}{2}\right)x^{-1/2-1}$

$= 35x^{-6} - 6x^{-3/2}$ Power function form

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(35x^{-6} - 6x^{-3/2}\right) = 35(-6)x^{-6-1} - 6\left(\frac{-3}{2}\right)x^{-\frac{3}{2}-1}$

$= -210x^{-7} + 9x^{-\frac{5}{2}}$ Power function form

$= \frac{-210}{x^7} + \frac{9}{x^{5/2}}$ Positive exponent form