Subject for this video:

Two Variable Applied Max Min Fence Problems

Reading: 4.6 Optimization

- **General:** Section ~~4.5~~ Absolute Maxima and Minima
- **More Specifically:** Pages 304 – 307 Examples 1 and 2

Homework: H66: Two Variable Applied Max Min Fence Problems (4.6#33,34*,35,36*)

In Section 4.6, we are studying problems involving Optimization.

Optimization problems are simply Max/Min problems, but they may have complications

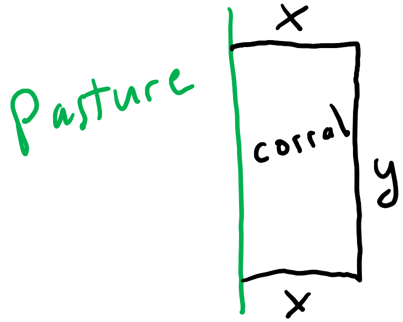
- They may be presented as word problems, about applications to real world situations.
- You will probably have to figure out the function and its domain
- They may have domains that are not closed intervals
- They may involve more than one variable

Homework H66 consists of

Two Variable Applied Max Min Fence Problems

[Example 1] (similar to 4.6#34*,35)

A farmer needs to build a fence to make a rectangular corral next to an adjacent pasture. He ~~needs~~ only needs to fence three sides, because the fourth side has already been fenced. He has 900 feet of fencing. What are the dimensions of the pasture that will enclose the largest possible area?



Equation I $\quad x + y + x = 900$

$\quad 2x + y = 900$

Equation II $\quad x \cdot y = A$ (area) $\quad$ maximize A.

Find positive numbers x, y such that the sum $2x + y = 900$

and the product $x \cdot y = A$ is maximized

This is exactly the abstract math problem that was solved in [Example 1] of the previous video

The result is that $x = 225$ (the sides perpendicular to pasture fence)

and $y = 450$ (The side parallel to pasture fence)

The corral will have area 101,250 square feet

Recall results from Video for H65 [Example 1] Find positive numbers x, y such that

- The sum $2x + y = 900$.
- The product maximized.

Solution:

(Step 1) Identify Equation I:

Result: $\boldsymbol{2x + y = 900}$

(Step 2) Write Equation II involving x and y and the letter P for the product.

Result $\boldsymbol{P = xy}$**. We want to maximize** $\boldsymbol{P}$**.**

(Step 3) Solve Equation I for y in terms of x.

Result: New Equation I: $\boldsymbol{y = 900 - x}$.

(Step 4) Substitute New Equation I into Equation II and simplify to get a new equation that gives the product A as a function of just one variable x. Call this function $P(x)$. Find domain.

Result: $\boldsymbol{P(x) = x(900 - 2x) = 900x - 2x^2}$**, with domain** $\boldsymbol{(0, 450)}$

(Step 5) Using Calculus, find the value of x that maximizes $P(x)$.

Result: $\boldsymbol{x = 225}$

(Step 6) Find corresponding values of y and the product.

Result: $\boldsymbol{y = 450, product = 101,250}$

[Example 2] (similar to 4.6#33,36*)

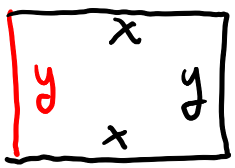
A farmer needs to build a rectangular fenced chicken yard. He wants an area of 9000 square feet.

The fence on the west side of the yard must be wood, which costs \$20/foot.

The fence on the east, north, south sides can be wire, which costs \$5/foot.

What are the dimensions of the cheapest fence? How much does it cost?

wood $\$20/\text{foot}$



wire $\$5/\text{foot}$

Equation I $\quad x \cdot y = 9000$

Cost C = North cost + South cost + East cost + West cost $= 5x + 5x + 5y + 20y$

Equation II $\quad C = 10x + 25y$

Find positive numbers x, y such that

the product $x \cdot y = 9000$

and the sum $2x + y = C$ is minimized

This is exactly the math problem solved in [Example 2] of previous video

The result was $x = 150$ (length of north & south sides)

$y = 60$ (length of east & west sides)

$C = 3000$ (cost of fence)

Recall results from Video for H65 [Example 2] Find positive numbers x, y such that

- The product is 9000.
- The sum $10x + 25y$ is minimized.

Solution:

(step 1) Write Equation I involving x and y expressing the fact that the product is 9000:

Result: $\boldsymbol{xy = 9000}$

(step 2) Write an equation II involving x and y and the letter S for sum:

Result $\boldsymbol{S = 10x + 25y}$

(step 3) Solve Equation I for y in terms of x. New Equation I.

Result: $\boldsymbol{y = 9000/x}$

(step 4) Substitute Equation I into Equation II and simplify to get a new equation that gives the sum S as a function of just one variable x. Call this function $S(x)$. Find its domain.

Result: $\boldsymbol{S(x) = 10x + 25(9000/x)}$ **with domain** $\boldsymbol{(0, \infty)}$

(step 5) Using calculus, find the value of x that minimizes $S(x)$.

Result: function.) $\boldsymbol{x = 150}$

(step 6) Find the corresponding values of y and sum, S.

Result: $\boldsymbol{y = 60}$ **and** $\boldsymbol{S = 3000}$