Subject for this video:

Introduction to Antiderivatives

Reading:

- **General:** Section 5.1 Antiderivatives and Indefinite Integrals
- **More Specifically:** Pages 322 – 324, Example 1

Homework:

H68: Is one function an antiderivative of another?

- Problems in MyLab: 5.1#35,37
- Problems in Book but not in MyLab 5.1#25,27,28,29,31,33,34,36,38

Capital F

small f

Definition of Antiderivative

Words: *F is an antiderivative of f.*

Meaning: f is the derivative of F. That is, $f = F'$.

Arrow diagram: $F \xrightarrow{\text{take derivative}} f$

[Example 1] Is $F(x) = \dfrac{x^3}{3}$ an antiderivative of $f(x) = x^2$?

Strategy: find $F'(x)$, see if it equals $f(x)$.

First, rewrite $F(x) = \dfrac{x^3}{3} = \left(\dfrac{1}{3}\right)x^3$

Now find the derivative

$$F'(x) = \frac{d}{dx}\left(\frac{1}{3}\right)x^3 = \left(\frac{1}{3}\right)\frac{d}{dx}x^3 = \frac{1}{\cancel{3}}\left(\cancel{3}x^{3-1}\right) = x^2 = f(x)$$

yes, because $F'(x) = f(x)$

[Example 2] Is $F(x) = \dfrac{(5x+7)^3}{3}$ an antiderivative of $f(x) = (5x+7)^2$?

Solution

Again, find $F'(x)$, see if $F'(x) = f(x)$

First rewrite $F(x)$ in power function form

$$F(x) = \frac{(5x+7)^3}{3} = \left(\frac{1}{3}\right)(5x+7)^3$$

$$F'(x) = \frac{d}{dx}\left(\frac{1}{3}\right)(5x+7)^3$$

$$= \left(\frac{1}{3}\right)\frac{d}{dx}(5x+7)^3$$

$$= \left(\frac{1}{3}\right)\frac{d}{dx}\,\text{outer}(\text{inner}(x))$$

chain rule

$$= \left(\frac{1}{3}\right)\text{outer}'(\text{inner}(x))\cdot\text{inner}'(x)$$

$$= \left(\cancel{\frac{1}{3}}\right)\cancel{3}(5x+7)^2\cdot 5 = (5x+7)^2\cdot 5 \neq f(x)$$

Chain Rule Details

$\text{inner}(x) = 5x+7$

$\text{inner}'(x) = 5$

$\text{outer}(\ ) = (\ \ )^3$

$\text{outer}'(\ ) = 3(\ \ )^2$

No, because $F'(x) \neq f(x)$

[Example 3] Is $F(x) = x\ln(x) - x$ an antiderivative of $f(x) = \ln(x)$?

Strategy: Find $F'(x)$, see if $F'(x) = f(x)$.

$$F'(x) = \frac{d}{dx}\left(x\ln(x) - x\right) =$$

$$= \underbrace{\frac{d}{dx}\left(x\ln(x)\right)}_{\text{Product Rule}} - \frac{d}{dx}x$$

$$= \left(\frac{d}{dx}x\right)\cdot\ln(x) + x\cdot\left(\frac{d}{dx}\ln(x)\right) - \frac{d}{dx}x$$

$$= (1)\cdot\ln(x) + \cancel{x}\cdot\left(\frac{1}{\cancel{x}}\right) - (1)$$

$$= \ln(x) + 1 - 1$$

$$= \ln(x)$$

$$= f(x)$$

Yes! Because $F'(x) = f(x)$.

[Example 4](A) Is $F(x) = e^{(x^2)}$ an antiderivative of $f(x) = e^{(x^2)}$?

Check by finding $F'(x)$

$$F'(x) = \frac{d}{dx} e^{(x^2)} = \frac{d}{dx} \text{outer}(\text{inner}(x))$$

$$\overset{\text{chain rule}}{=} \text{outer}'(\text{inner}(x)) \cdot \text{inner}'(x)$$

$$= e^{(x^2)} \cdot 2x \neq f(x)$$

No, not an antiderivative, because $F'(x) \neq f(x)$.

Chain Rule Details

$\text{inner}(x) = x^2$

$\text{inner}'(x) = 2x$

$\text{outer}(\ ) = e^{(\ )}$

$\text{outer}'(\ ) = e^{(\ )}$

(B) Is $F(x) = e^{\left(\frac{x^3}{3}\right)}$ an antiderivative of $f(x) = e^{(x^2)}$?

Check by Finding $F'(x)$

$$F'(x) = \frac{d}{dx}\left(e^{\left(\frac{x^3}{3}\right)}\right) = \frac{d}{dx}\left(\text{outer}(\text{inner}(x))\right)$$

$$\overset{\text{chain rule}}{=} \text{outer}'(\text{inner}(x)) \cdot \text{inner}'(x)$$

$$= e^{\left(\frac{x^3}{3}\right)} \cdot x^2$$

$$\neq f(x)$$

No, not an antiderivative, because $F'(x) \neq f(x)$.

Chain Rule Details

$\text{inner}(x) = \frac{x^3}{3} = \left(\frac{1}{3}\right)x^3$

$\text{inner}'(x) = \left(\frac{1}{3}\right)3x^2 = x^2$

$\text{outer}(\ ) = e^{(\ )}$

$\text{outer}'(\ ) = e^{(\ )}$

Important fact from higher math:

The Good News: The function $f(x) = e^{(x^2)}$ does have an antiderivative.

The Bad News: That antiderivative cannot be written in the usual way as a function made up of basic functions. That is, it cannot be expressed as a finite combination of simple functions.

Therefore, when presented with the question

$$\text{Is } F(x) = \textit{some ordinary function} \text{ an antiderivative of } f(x) = e^{(x^2)}?$$

The answer will always be *no*. But one must always justify that answer by finding $F'(x)$ and confirming that $F'(x) \neq f(x)$.

[Example 5] True/False Questions

(A) The constant function $f(x) = \pi$ is an antiderivative of the constant function $k(x) = 0$. T F

Strategy: Find $f'(x)$, see if it equals $k(x)$.

$$f'(x) = \frac{d}{dx}(\pi) = 0 = k(x).$$

constant function

(B) the constant function $k(x) = 0$ is an antiderivative of the constant function $f(x) = \pi$. T F

Find $k'(x) = \frac{d}{dx} k(x) = \frac{d}{dx} 0 = 0 \neq f(x)$

(C) The constant function $k(x) = 0$ is an antiderivative of itself. T F

True, because $k'(x) = \frac{d}{dx} k(x) = \frac{d}{dx}(0) = 0 = k(x)$

(D) The function $g(x) = 5e^{(x)}$ is an antiderivative of itself. T F

Observe $g'(x) = \frac{d}{dx} 5e^{(x)} = 5 \frac{d}{dx} e^{(x)} = 5 \cdot e^{(x)} = g(x)$

constant multiple rule

exponential function rule #1

(E) The function is $h(x) = 5e^{\pi}$ an antiderivative of itself. T F

$$h'(x) = \frac{d}{dx} h(x) = \frac{d}{dx} \underline{5e^{\pi}} = 0 \neq h(x)$$

constant function

[Example 6] For which values of n is $F(x) = \frac{x^{n+1}}{n+1}$ is an antiderivative of $f(x) = x^n$?

We need $F'(x)$

Start by rewriting $F(x) = \frac{x^{n+1}}{n+1} = \left(\frac{1}{n+1}\right) x^{n+1}$

power function form

So $F'(x) = \frac{d}{dx}\underbrace{\left(\frac{1}{n+1}\right)}x^{n+1} = \underbrace{\left(\frac{1}{n+1}\right)}\frac{d}{dx}x^{n+1} = \cancel{\left(\frac{1}{n+1}\right)} \cdot \left(\cancel{(n+1)}x^{(n+1)-1}\right)$

constant multiple rule

power rule

$= x^n$

$= f(x)$

Observe: When $n=-1$, $F(x) = \frac{x^{-1+1}}{-1+1} = \frac{x^0}{0}$ Does not exist!

So for all real numbers $n \neq -1$,
$F(x)$ will be an antiderivative of $f(x)$.