Subject for this video:

The Collection of All Antiderivatives of a Function

Reading:

- **General:** Section 5.1 Antiderivatives and Indefinite Integrals
- **More Specifically:** Pages 322 – 324, Example 1

Homework:

H69: Graphs of antiderivatives of a function (5.1: #39,41*)

Recall the definition of *antiderivative* from the previous video.

> **Definition of Antiderivative**
>
> **Words:** *F is an antiderivative of f.*
>
> **Meaning:** f is the derivative of F. That is, $f = F'$.
>
> **Arrow diagram:** $F \xrightarrow{\text{take derivative}} f$

And recall this example from the previous video:

[Example 1] Is $F(x) = \dfrac{x^3}{3}$ an antiderivative of $f(x) = x^2$?

We found

$$F'(x) = \frac{d}{dx}F(x) = \frac{d}{dx}\left(\frac{x^3}{3}\right) = \cdots = x^2 = f(x)$$

$F(x)$ is an antiderivative of $f(x)$ because $F'(x) = f(x)$.

[Example 1](continued) Is $G(x) = \frac{x^3}{3} + 17$ an antiderivative of $f(x) = x^2$?

check

$$G'(x) = \frac{d}{dx}\left(\frac{x^3}{3} + 17\right) = \left(\frac{d}{dx}\frac{x^3}{3}\right) + \frac{d}{dx}17 = x^2 + 0 = x^2 = f(x)$$

constant function

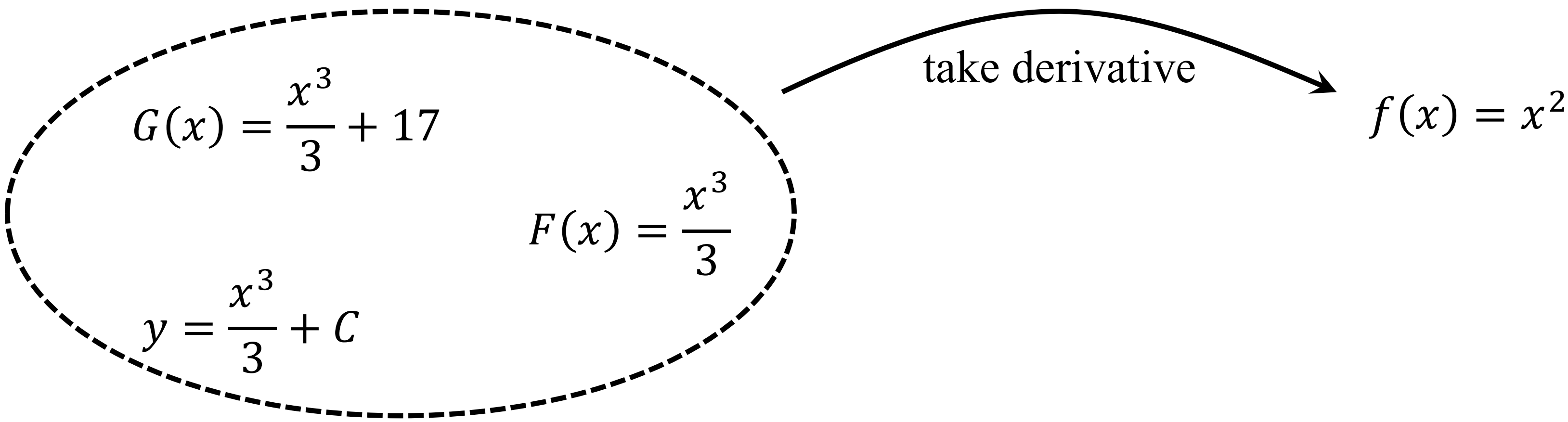
by result of earlier example.

So $G(x) = \frac{x^3}{3} + 17$ <u>is</u> an antiderivative of $f(x) = x^2$

We see from **[Example 1]** that there is more than one antiderivative of $f(x)$, and we realize that there are many more than just the two that we have seen in this example. Any function of the form

$$y = \frac{x^3}{3} + C$$

where C is a real number) will also be an antiderivative of $f(x)$.



the collection of all antiderivatives of $f(x)$

This realization lets us appreciate a subtlety in the definition of *antiderivative*.

Notice that the definition uses the sentence

*F is **an** antiderivative of f.*

It does not say

*F is **the** antiderivative of f.*

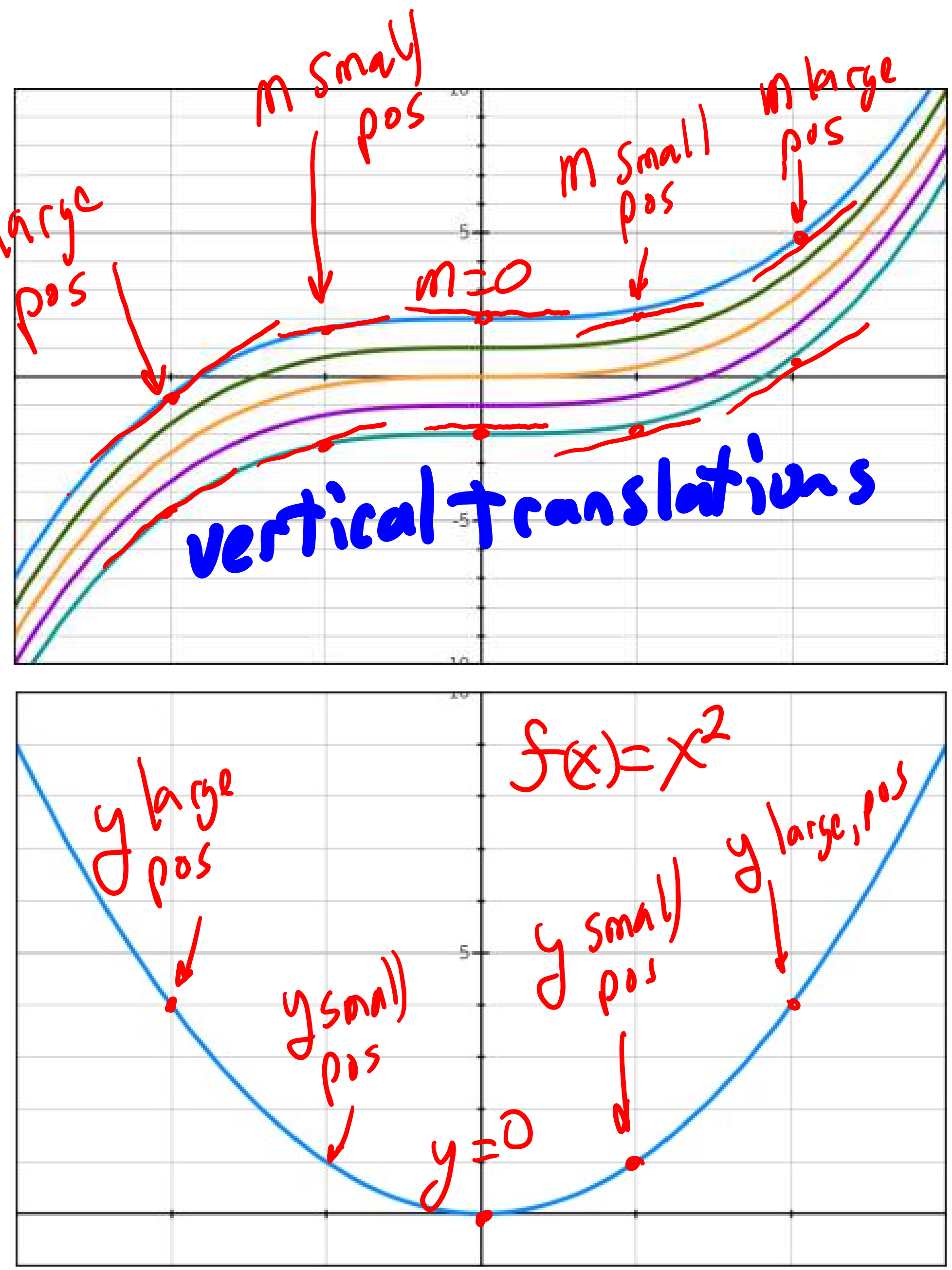
Now we see why. There is not just one antiderivative of f; there are many!

This makes sense in terms of the graphs of functions of the form $F(x) + C$

$y = \frac{x^3}{3} + C$

tangent lines on all these graphs are parallel, with the same slope

find the derivative



$y = \frac{x^3}{3} + 1$

$y = \frac{x^3}{3}$

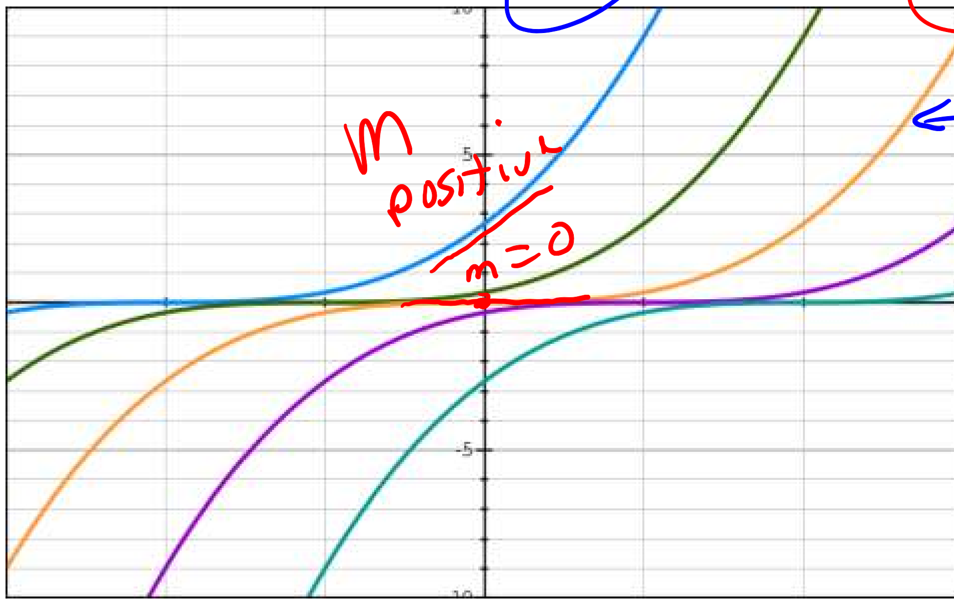
the numbers that are the slopes of the tangent lines

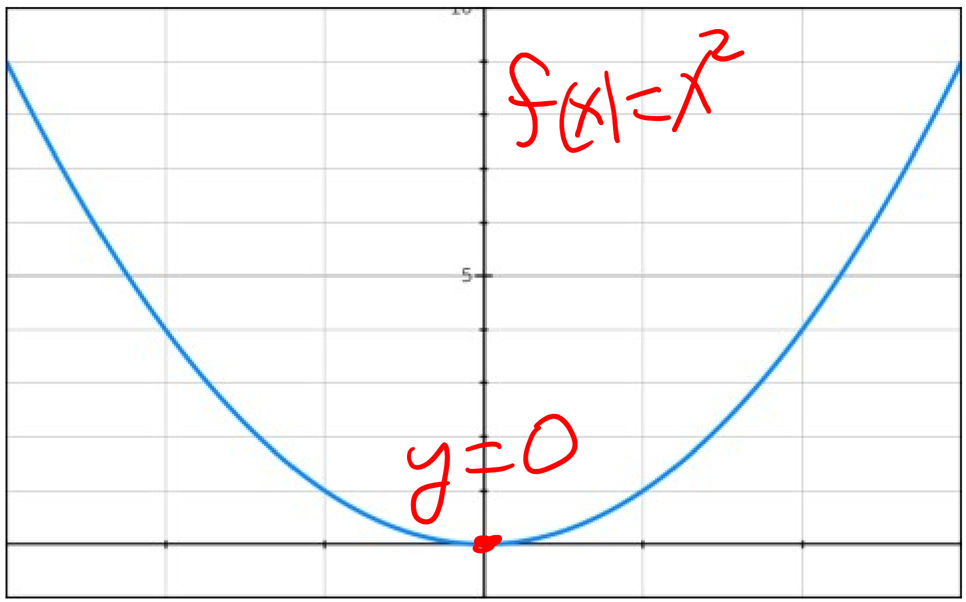
equal

the numbers that are the y values on this graph

Note that horizontal translations of $F(x) = x^3/3$ are *not* antiderivatives of $f(x) = x^2$

tangent lines are not parallel





The observations from **[Example 1]** can be generalized to other functions.

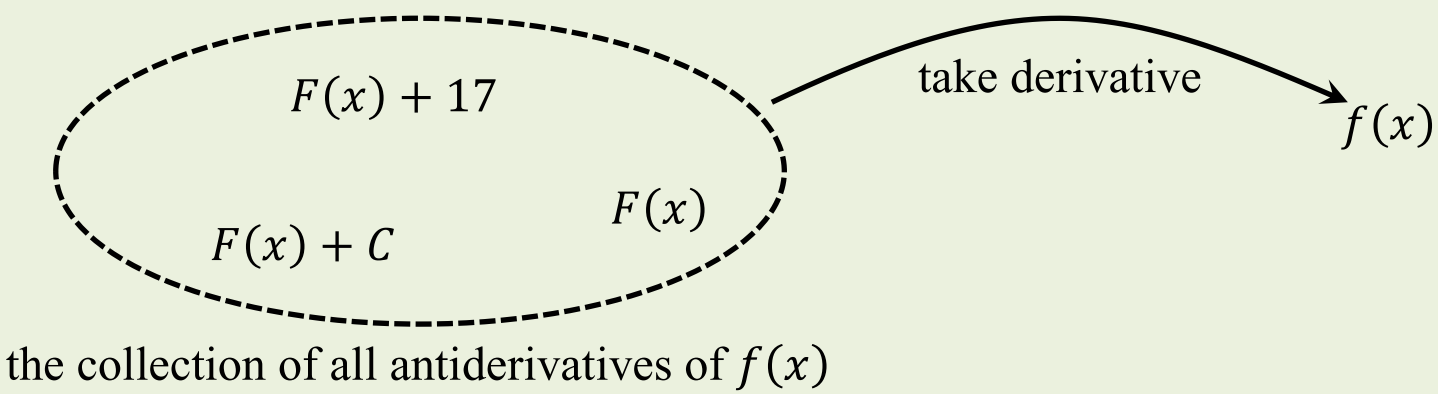
Theorem about the Collection of Antiderivatives of a Function

If a function $F(x)$ is an antiderivative of a function $f(x)$, then any function of the form

$$F(x) + C$$

where C is a real number constant, will also be an antiderivative of $f(x)$.

Furthermore, these are *all* the antiderivatives of $f(x)$. That is, if $G(x)$ is also an antiderivative of $f(x)$, then it must be that $G(x) = F(x) + C$ where C is a real number constant.



Particular Antiderivative and General Antiderivative

Return to [Example 1], in which we observed that, for the function $f(x) = x^2$,

- The function $F(x) = \frac{x^3}{3}$ is an antiderivative of $f(x)$.
- The function $G(x) = \frac{x^3}{3} + 17$ is also an antiderivative of $f(x)$.
- Any function of the form $y = \frac{x^3}{3} + C$ where C is a real number, is an antiderivative of $f(x)$.

When a choice of an *actual number* for C is made, the resulting function is called a ***particular antiderivative*** of $f(x)$. That is,

- The function $F(x) = \frac{x^3}{3}$ is a *particular antiderivative* of $f(x)$.
- The function $G(x) = \frac{x^3}{3} + 17$ is a *particular antiderivative* of $f(x)$.

But if C has not been chosen, then the *function form*

$$y = \frac{x^3}{3} + C$$

is called ***the general antiderivative*** of $f(x)$. Note the use of the word ***the***. Realize that this expression is a *function form*. There is only one such form.

Even this is subtle. The general antiderivative of $f(x)$ can appear in different guises. For instance, the function form

$$y = \frac{x^3}{3} + C$$

where C is a constant that can be any real number, is the *general antiderivative* of $f(x) = x^2$. But the function form

$$y = \frac{x^3}{3} + 17 + D$$

where D is a constant that can be any real number, is also the *general antiderivative* of $f(x) = x^2$. This will be confusing at first. The key is to realize that the two expressions above are *the same form*. That is, if D can be any real number, then the quantity $17 + D$ can also be any real number. So having a $+17 + D$ tacked onto the end is the same as having a $+C$ tacked onto the end.