Subject for this video:

Find Particular Antiderivative Satisfying Extra Condition

Reading:

- **General:** Section 5.1 Antiderivatives and Indefinite Integrals
- **More Specifically:** Page 330, Example 4

Homework: H72: Find particular antiderivative satisfying extra condition (5.1#55, 57,58, 61)

Recall the definition of *antiderivative* from previous videos.

Definition of Antiderivative

Words: *F is an antiderivative of f.*

Meaning: f is the derivative of F. That is, $f = F'$.

Arrow diagram: $F \xrightarrow{\text{take derivative}} f$

Theorem about the Collection of Antiderivatives of a Function

If a function $F(x)$ is an antiderivative of a function $f(x)$, then any function of the form $F(x) + C$ where C is a real number constant, will also be an antiderivative of $f(x)$. Furthermore, these are *all* the antiderivatives of $f(x)$. That is, if $G(x)$ is also an antiderivative of $f(x)$, then it must be that $G(x) = F(x) + C$ where C is a real number constant.

$$\{\,F(x) + 17,\quad F(x),\quad F(x) + C\,\} \xrightarrow{\text{take derivative}} f(x)$$

the collection of all antiderivatives of $f(x)$

Recall the definition of *particular antiderivative* and *general antiderivative*

When a choice of an *actual number* for C is made, the resulting function is called a ***particular antiderivative*** of $f(x)$. That is,

- The function $F(x) = \frac{x^3}{3}$ is a *particular antiderivative* of $f(x) = x^2$.
- The function $G(x) = \frac{x^3}{3} + 17$ is a *particular antiderivative* of $f(x) = x^2$.

But if C has not been chosen, then the *function form*

$$y = \frac{x^3}{3} + C$$

is called ***the general antiderivative*** of $f(x)$.

Recall the definition of *indefinite integral*

Definition of Indefinite Integral

symbol: $\int f(x)dx$

spoken: the ***indefinite integral*** of $f(x)$

meaning: the ***general antiderivative*** of $f(x)$

Remark: We know that, given one function $F(x)$ that is known to be an antiderivative of $f(x)$, we can get all other antiderivatives by adding constants to $F(x)$. The *general antiderivative* of $f(x)$ is denoted by writing $F(x) + C$, where C is a constant that can be any real number. That is,

$$\text{If } F'(x) = f(x) \text{ then } \int f(x)dx = F(x) + C$$

Additional Terminology:

- The function $f(x)$ inside the integral symbol is called the ***integrand***.
- The unknown constant C is called the ***constant of integration***.

And finally, recall the *Indefinite Integral Rules* that we discussed in the previous video, presented here along with two new rules.

The *power rule*:	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ when $n \neq -1$
The $\frac{1}{x}$ *rule*:	$\int \frac{1}{x} dx = \ln(\lvert x \rvert) + C$ for all $x \neq 0$
The $e^{(x)}$ *rule*:	$\int e^{(x)} dx = e^{(x)} + C$
The $\ln(x)$ *rule*:	$\int \ln(x)\, dx = x \ln(x) - x + C$
The *constant multiple rule*:	$\int af(x)dx = a \int f(x)dx$
The *sum rule*:	$\int f(x) \pm g(x)dx = \int f(x)dx \pm \int g(x)dx$
The *sum and constant multiple rule*:	$\int af(x) \pm bg(x)dx = a \int f(x)dx \pm b \int g(x)dx$

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So far in Section 5.1., we have often used the following conventions when discussing functions and their antiderivatives:

- We have used a lowercase letter for a function
- We have used an uppercase letter for an antiderivative or indefinite integral of that function

Suppose:

$$F(x) \xrightarrow{\text{take derivative}} f(x)$$

an antiderivative
of $f(x)$

Then:

$$\begin{tikzcd} \int f(x)dx = F(x) + C \arrow[r, bend left, "\text{take derivative}"] & f(x) \arrow[l, bend left, "\text{find indefinite integral}"] \end{tikzcd}$$

general
antiderivative
of $f(x)$

In this video, and in the accompanying homework set, we will use a different style of notation.

We will start by knowing $f'(x)$, and we will seek to find $f(x)$.

This situation can be illustrated by slightly modifying the previous diagram.

Suppose: Derivative $f'(x)$ is given.

Find: General antiderivative $f(x) + C$ such that

$$\begin{tikzcd} \int f'(x)dx = f(x) + C \arrow[rr, bend left, dashed, "\text{take derivative}"] & & f'(x) \arrow[ll, bend left, "\text{find indefinite integral}"] \end{tikzcd}$$

general
antiderivative
of $f'(x)$

[Example 1] (a) Find the general antiderivative of the derivative $C'(x) = 12x^2 - 22x$.

Solution

General antiderivative $= \int C'(x)\,dx = \int 12x^2 - 22x\,dx =$

constant multiple rule

$n=2$ $n=1$

$= 12\int x^2\,dx - 22\int x\,dx$

power rule

$= 12\left(\frac{x^{2+1}}{2+1} + D\right) - 22\left(\frac{x^{1+1}}{1+1} + E\right)$

$= 12\left(\frac{x^3}{3} + D\right) - 22\left(\frac{x^2}{2} + E\right)$

$= \frac{12x^3}{3} + 12D - \frac{22x^2}{2} - 22E$

$= 4x^3 - 11x^2 + 12D - 22E$

$C(x) = 4x^3 - 11x^2 + K$ where K can be any real number

(b) (similar to 5.1#55) Find the particular antiderivative of the derivative $C'(x) = 12x^2 - 22x$ that satisfies $C(0) = 30$.

extra condition.

Strategy Find the General Antiderivative

We did this in (a). The result was

$$C(x) = 4x^3 - 11x^2 + K$$

Find the value of the constant of integration that will satisfy the extra condition.

$$30 = C(0) = 4(0)^3 - 11(0)^2 + K = 0 + K = K$$

So $K = 30$

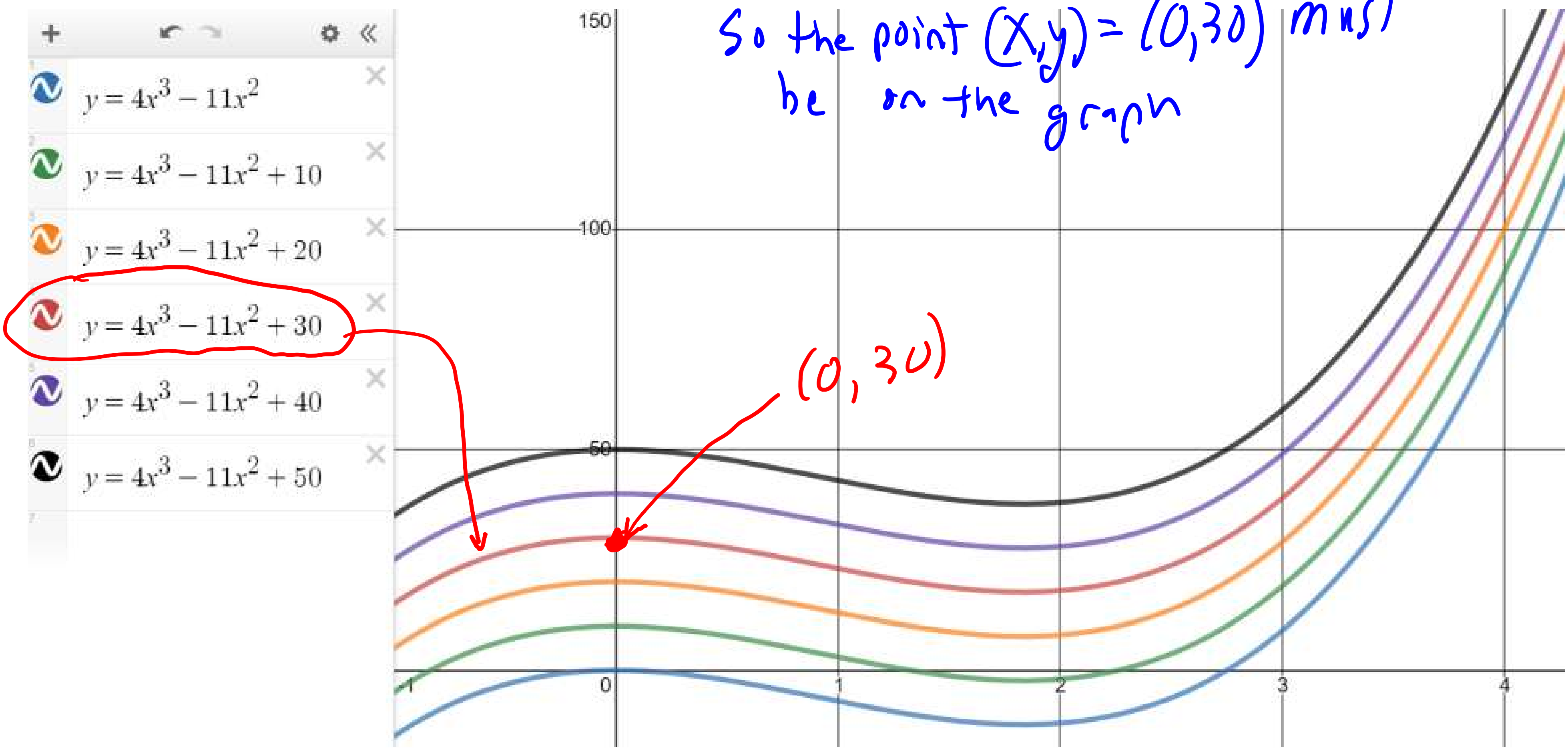
Write clear conclusion presenting the Particular Antiderivative

$$C(x) = 4x^3 - 11x^2 + 30$$

(c) Illustrate your result on the given collection of graphs.

must have $C(0)=30$

So the point $(x,y)=(0,30)$ must be on the graph



[Example 2] (similar to 5.1#5)

(a) Find f such that

$$f'(x) = \frac{7}{\sqrt{x}} \quad and\ f(25) = 80$$

Integrate to find the general antiderivative

Rewrite $f'(x) = \frac{7}{\sqrt{x}} = \frac{7}{x^{1/2}} = 7 \cdot x^{-1/2}$

power function form

Now integrate

$$f(x) = \int 7x^{-1/2}dx \underset{\substack{\uparrow \\ \text{constant multiple rule}}}{=} 7\int x^{-1/2}dx \underset{\substack{\uparrow \\ \text{power rule}}}{=} 7\left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C\right) = 7\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C\right)$$

$$= 7(2\sqrt{x} + C) = 14\sqrt{x} + 7C = \underbrace{14\sqrt{x} + D}_{\text{general antiderivative}}$$

Find value of the constant of integration to satisfy the extra condition

$$80 = f(25) = 14\sqrt{25} + D = 14(5) + D = 70 + D$$

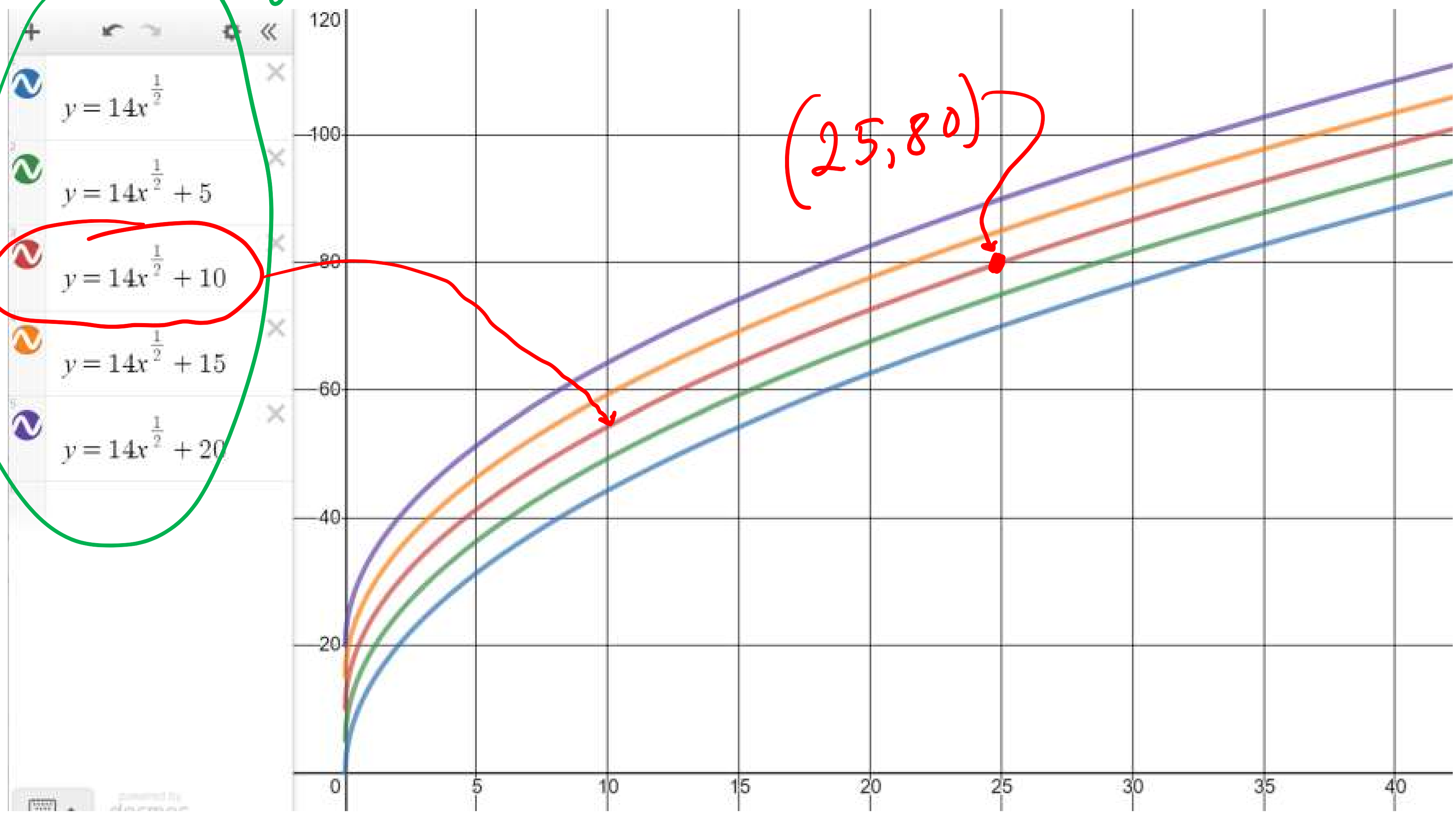
So $D = 10$

Write clear conclusion presenting the particular antiderivative

$$f(x) = 14\sqrt{x} + 10$$

(b) Illustrate your result on the given collection of graphs.

← general antiderivative is $f(x) = 14\sqrt{x} + D$



$f(25) = 80$ means $\underline{(x, y) = (25, 80)}$ is on the graph.

[Example ~~14~~ 3] (a) (similar to 5.1#61) Find the particular antiderivative of the derivative

$$\frac{df}{dx} = 5e^{(x)} - 19$$

that satisfies $f(0) = 25$. extra condition

Power function $1 = x^0$

Integrate to find the general antiderivative

First rewrite the integrand $\frac{df}{dx} = 5e^{(x)} - 19 = 5e^{(x)} - 19 \cdot 1$

Now integrate

$$f(x) = \int \frac{df}{dx}\,dx = \int 5e^{(x)} - 19\cdot 1\,dx = 5\int e^{(x)}dx - 19\int 1\,dx$$

Sum and constant multiple rule

Power rule with $n=0$.

$$= 5\left(e^{(x)} + C\right) - 19\left(\frac{x^{0+1}}{0+1} + D\right)$$

$$= 5e^{(x)} + 5C - 19\frac{x^1}{1} - 19D$$

$$= 5e^{(x)} - 19x + 5C - 19D$$

$$f(x) = 5e^{(x)} - 19x + E$$ general antiderivative

Satisfy the extra condition $f(0) = 25$

$$25 = f(0) = 5e^{(0)} - 19(0) + E$$

$$= 5 \cdot 1 - 0 + E$$

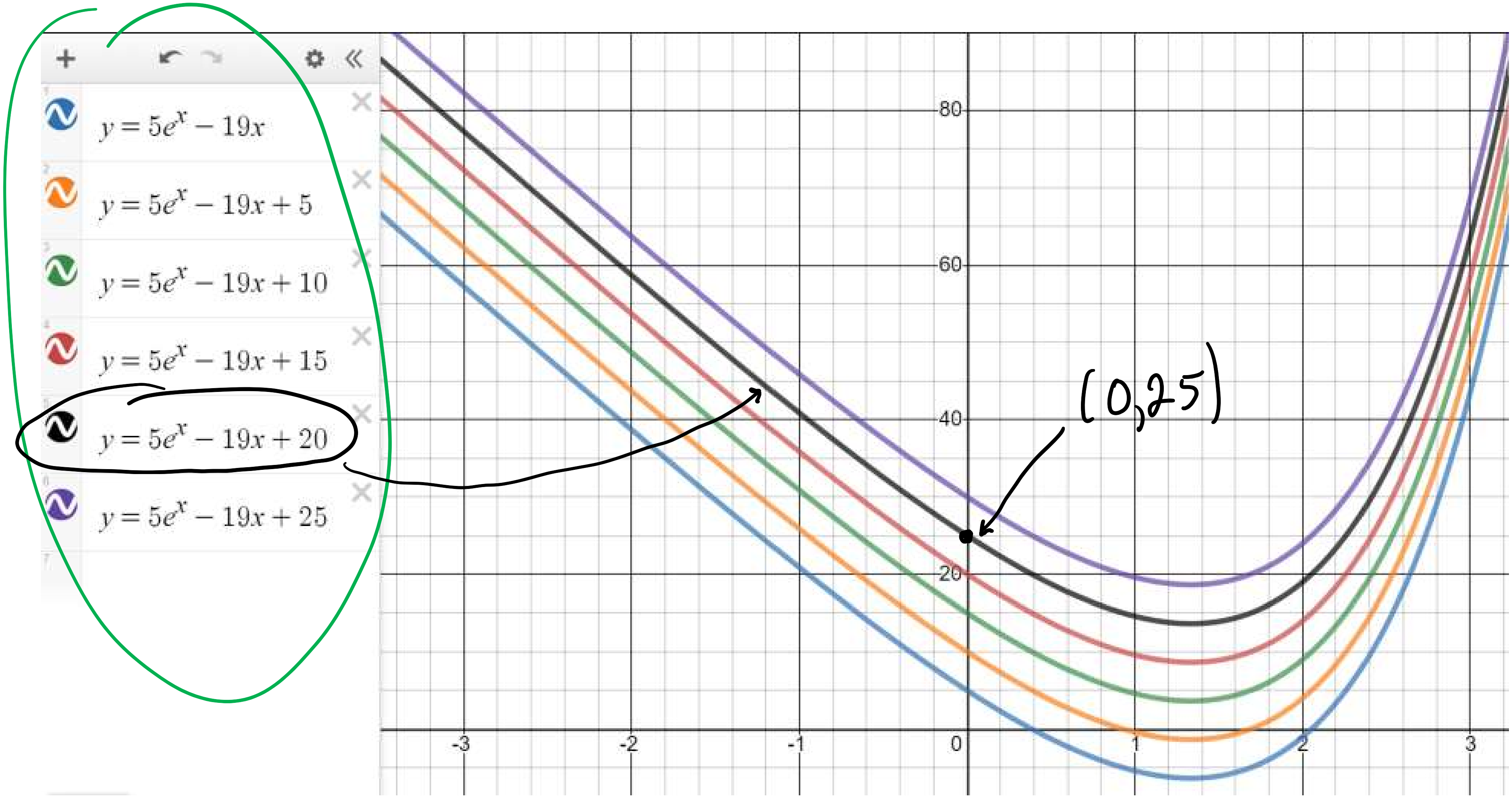
$$25 = 5 + E$$

$$20 = E$$

Write Clear Conclusion presenting the particular antiderivative

$$f(x) = 5e^{(x)} - 19x + 20$$ particular antiderivative

(b) Illustrate your result on the given collection of graphs.



Extra condition $f(0)=25$ means that $(x,y)=(0,25)$ is on graph

(c) Now change variable to t and the problem statement becomes

Find the particular antiderivative of the derivative

$$\frac{df}{dt} = 5e^{(t)} - 19$$

that satisfies $f(0) = 25$.

Result $f(t) = 5e^{(t)} - 19t + 20$

particular antiderivative

(d) Now change function name to $x(t)$, and we get a problem statement similar to that of 5.1#61.

Find the particular antiderivative of the derivative

$$\frac{dx}{dt} = 5e^{(t)} - 19$$

that satisfies $x(0) = 25$.

Result

$$x(t) = 5e^{(t)} - 19t + 20$$

particular antiderivative