Subject for this video:

Introduction to Integration by Substition

Reading:

- **General:** Section 5.2 Integration by Substitution
- **More Specifically:** Page 335 - 343, but no exact match of Examples

Homework: H74: Basic Substitution Integrals: No Multiplicative Constant after Substitution (5.2#11,15,17,19,67)

Remember that the *Chain Rule for Derivatives* is used for taking the *derivative* of *nested functions*:

Chain Rule for Derivatives: $\frac{d}{dx} outer\big(inner(x)\big) = outer'\big(inner(x)\big) \cdot inner'(x)$

The goal now is to find the *general antiderivative* of a function $f(x)$ that involves a *nested function*. That is, we wish to find the *indefinite integral*

$$\int f(x)\,dx$$

where the integrand $f(x)$ involves a nested function.

This is not always possible. But sometimes it is, using a method called the *Substitution Method*.

The Substitution Method for finding the *indefinite integral*

$$F(x) = \int f(x)\, dx$$

where the integrand $f(x)$ involves a *nested function*.

Step 1 Identify the inner function and call it u. Write the equation $inner(x) = u$ to introduce the single letter u to represent the inner function. Circle the equation.

Step 2 Build the equation $dx = \frac{1}{u'}du$. To do this, first find u', then use it to build equation $dx = \frac{1}{u'}du$. Circle the equation.

Step 3 Substitute, Cancel, Simplify. In steps (1) and (2) you have two circled equations. Substitute these into the integrand of your indefinite integral. Cancel as much as possible and simplify by using the *Constant Multiple Rule*. The result should be a new basic integral involving just the variable u.

Step 4 Integrate. Find the new indefinite integral by using the indefinite integral rules. The result should be a function involving just the variable u (with constant of integration $+C$).

Step 5 Substitute Back. Substitute $u = inner(x)$ into your function from Step (4) The result will be a new function of just the variable x. (Be sure to include the constant of integration $+C$ in your result.) This is the $F(x)$ that we seek.

Note that the result of **Step 3** should be a new indefinite integral with an integrand that is a function involving the variable u. There are three important things to check at the end of **Step 3**:

- There should be no x in the new indefinite integral. It should involve only u.
- The new indefinite integral should *not* involve a *nested function*, and it should be a *basic integral* that can be integrated using our indefinite integral rules.
- If the above two items are not satisfied, then either you made a mistake, or the original integral might be one for which the Substitution Method cannot be used.

[Example 1] Find the indefinite integrals

(***A***) $\int 6x^5(x^6+1)^4 dx$ integrand $f(x) = 6x^5(x^6+1)^4$

Step 1. Identify the inner function and call it u.

$$x^6+1=u$$

Step 2. Build the equation $dx = \frac{1}{u'}du$

First get u' $u = x^6+1$

so $u' = \frac{du}{dx} = 6x^5$

Now Build equation $dx = \frac{1}{6x^5}du$

Step 3. Substitute, Cancel, Simplify

$$\int 6x^5(x^6+1)^4 dx \underset{\uparrow \text{ Substitute}}{=} \int \cancel{6x^5}(u)^4\left(\frac{1}{\cancel{6x^5}}du\right) \underset{\uparrow \text{ cancel}}{=} \int u^4 du$$

(no simplifying needs to be done)

Step 4. Integrate $\int u^4 du = \frac{u^{4+1}}{4+1} + C = \frac{u^5}{5} + C$

Step 5. Substitute back $\frac{u^5}{5} + C = \frac{(x^6+1)^5}{5} + C = F(x)$ the general antiderivative

Conclusion $\int 6x^5(x^6+1)^4 dx = \frac{(x^6+1)^5}{5} + C$

(B) $\int \sqrt{1+x^4}(4x^3)dx$

The integrand is typeset in a misleading way. (The book uses this weird typesetting)
Start by rewriting the integrand in a more helpful way.

$$\text{Integrand} = f(x) = \sqrt{1+x^4}\,(4x^3) = (1+x^4)^{1/2}\cdot 4x^3$$

$$\int (1+x^4)^{1/2}\, 4x^3\, dx$$

<u>Step 1</u> <u>Identify</u> the inner function and call it u. $1+x^4 = u$

<u>Step 2</u> <u>Build the equation</u> $dx = \frac{1}{u'}du$ get u'

$u = 1+x^4$

$u' = \frac{du}{dx} = 4x^3$

build equation $dx = \frac{1}{4x^3}du$

<u>Step 3</u> <u>Substitute, Cancel, Simplify</u>

$$\int (1+x^4)^{1/2}\cdot 4x^3\, dx \underset{\text{Substitute}}{=} \int (u)^{1/2}\cdot \cancel{4x^3}\,\frac{1}{\cancel{4x^3}}du \underset{\text{cancel}}{=} \int u^{1/2}du$$

no further Simplifying needed

<u>Step 4</u> <u>Integrate</u>

$$\int u^{1/2}du \underset{\text{power rule}}{=} \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{u^{\frac{3}{2}}}{3/2} + C = \frac{2u^{3/2}}{3} + C$$

<u>Step 5</u> <u>Substitute back</u>

$$\boxed{\int (1+x^4)^{1/2}\, 4x^3\, dx = \frac{2(1+x^4)^{3/2}}{3} + C}$$

(C) $\int \frac{(\ln(x))^{13}}{x}\,dx = \int (\ln(x))^{13} \cdot \frac{1}{x}\,dx$

Step 1 Identify the inner function and call it u

$\ln(x) = u$

Step 2 Build the equation $dx = \frac{1}{u'}du$

Find u': $u = \ln(x)$

$u' = \frac{du}{dx} = \frac{d}{dx}\ln(x) = \frac{1}{x}$

Build the equation $dx = \frac{1}{\frac{1}{x}}du$

$dx = x\,du$

Step 3 Substitute, Cancel, Simplify

$\int (\ln(x))^{13} \cdot \frac{1}{x}\,dx \underset{\text{substitute}}{=} \int (u)^{13} \cdot \frac{1}{x}\,x\,du \underset{\text{cancel}}{=} \int u^{13}\,du$ no further simplification needed

Step 4 Integrate $\int u^{13}\,du = \frac{u^{13+1}}{13+1} + C = \frac{u^{14}}{14} + C$

Step 5 Substitute back

$\boxed{\int \frac{(\ln(x))^{13}}{x}\,dx = \frac{(\ln(x))^{14}}{14} + C}$

(**D**) $\int \frac{1}{5x-7}(5)\,dx$ $=$ $\int \frac{1}{(5x-7)} \cdot 5\,dx$

written in more helpful way

Step 1 Identify the inner function and call it u $\quad 5x-7=u$

Step 2 Build the equation $dx = \frac{1}{u'}du$

First find u'

$$u = 5x-7$$

$$u' = \frac{du}{dx} = \frac{d}{dx}(5x-7) = 5$$

Build the equation $\quad dx = \frac{1}{5}du$

Step 3 Substitute, Cancel, Simplify

$$\int \frac{1}{5x-7} \cdot 5\,dx = \int \frac{1}{u} \cdot 5 \cdot \frac{1}{5}du = \int \frac{1}{u}du$$

Substitute $\qquad$ Cancel $\qquad$ no further simplification needed

Step 4 Integrate

$$\int \frac{1}{u}du = \ln(|u|) + C$$

Step 5 Substitute Back

$$\int \frac{1}{5x-7} \cdot 5\,dx = \ln(|5x-7|) + C$$

(E) $\int e^{(x^3)}(3x^2)dx = \int e^{(x^3)} \cdot 3x^2 dx$

Step 1 Identify the Inner Function and Call it u

$x^3 = u$

Step 2 Build the equation $dx = \frac{1}{u'} du$

First find $u' = \frac{du}{dx} = \frac{d}{dx} x^3 = 3x^2$

Use u' to build the equation

$dx = \frac{1}{3x^2} du$

Step 3 Substitute, Cancel, Simplify

$$\int e^{(x^3)} \cdot 3x^2 dx \underset{\substack{\uparrow \\ \text{Substitute}}}{=} \int e^{(u)} \cdot 3x^2 \frac{1}{3x^2} du \underset{\substack{\uparrow \\ \text{cancel}}}{=} \int e^{(u)} du$$

no further simplifying needed.

Step 4 Integrate

$$\int e^{(u)} du = e^{(u)} + C$$

Step 5 Substitute Back

$$\int e^{(x^3)} \cdot 3x^2 dx = e^{(x^3)} + C$$